Reasoning About Dynamical Systems

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- possibility of interaction with other components during overall computation,
- observable through well-defined interfaces to ensure flow of data.
- often acting as a 'building blocks' of larger, concurrent, systems

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an *observation structure*: universe $\xrightarrow{p}$ $\bigcirc\!\frown\!\bigcirc$ universe

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(an interface T)

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(a system of type T)

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where

$$\begin{aligned} \mathsf{at}_\omega\ (\phi, i) &= \phi\ i \\ \mathsf{m}_\omega\ (\phi, i) &= \lambda\, s\ .\ \phi < i : s > \end{aligned}$$

Relating Systems

$h : \langle U, \alpha \rangle \longrightarrow \langle U', \alpha' \rangle$ is a function $h : U \longrightarrow U'$ such that

$$\begin{CD} U @>{\alpha}>> \mathsf{T}U \\ @V{h}VV @VV{\mathsf{T}\ h}V \\ U' @>{\alpha'}>> \mathsf{T}U' \end{CD}$$

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- e.g.

$$\begin{array}{ccc} U \times I & \xrightarrow{\langle \mathsf{at}, \mathsf{m} \rangle} & O \times U \\ {\scriptstyle h \times \mathsf{id}}\downarrow & & \downarrow{\scriptstyle \mathsf{id} \times h} \\ U' \times I & \xrightarrow{\langle \mathsf{at}', \mathsf{m}' \rangle} & O \times U' \end{array}$$

$$\begin{aligned} \mathsf{at} &= \mathsf{at}' \cdot (h \times \mathsf{id}) \\ h \cdot \mathsf{m} &= \mathsf{m}' \cdot (h \times \mathsf{id}) \end{aligned}$$

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\downarrow{\scriptstyle \mathsf{bh} \times \mathsf{id}} & & \downarrow{\scriptstyle \mathsf{id} \times \mathsf{bh}} \\
O^{I^+} \times I & \xrightarrow{\langle \mathsf{at}_\omega, \mathsf{m}_\omega \rangle} & O \times O^{I^+}
\end{array}
$$

What's special about $\langle O^{I^+}, \overline{\langle \mathsf{at}_\omega, \mathsf{m}_\omega \rangle} \rangle$?

- There is always a morphism — bh — to it from any $\langle U, \overline{\langle \mathsf{at}, \mathsf{m} \rangle} \rangle$

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This system is itself unique up to isomorphism and can be characterized by an universal property: finality.

Going Generic: functors and coalgebras

Functors, Coalgebras, Seeds & Behaviours

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(a T-coalgebra)

A functor is an uniform transformation of sets and functions, which preserves identities and composition.

Final Coalgebras and Anamorphisms

$$\begin{tikzcd} \nu_{\mathsf{T}} \arrow[r, "\omega_{\mathsf{T}}"] & \mathsf{T}\ \nu_{\mathsf{T}} \\ U \arrow[u, "[\![(p)]\!]_{\mathsf{T}}"] \arrow[r, "p"] & \mathsf{T}\ U \arrow[u, "\mathsf{T}\ [\![(p)]\!]_{\mathsf{T}}"'] \end{tikzcd}$$

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whose commutativity equivales to the following universal law:

$$k = [\![(p)]\!]_{\mathsf{T}} \;\Leftrightarrow\; \omega_{\mathsf{T}} \cdot k = \mathsf{T}\ k \cdot p$$

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$\boxed{\text{Clearly, } [\![(p)]\!]_{\mathsf{T}} = \text{bh}}$

What universality means

- Existence $\equiv$ definition principle

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[\![(\omega_{\mathsf{T}})]\!] &= \mathsf{id}_{\nu_{\mathsf{T}}} \\
[\![(p)]\!] \cdot h &= [\![(q)]\!] \quad \text{if} \quad p \cdot h = \mathsf{T}\,h \cdot q
\end{aligned}
$$

Coinductive Definition

Stream Generation

$$\begin{tikzcd}
A^\omega \arrow[r, "\langle \mathsf{hd},\mathsf{tl}\rangle"] & A \times A^\omega \\
A \arrow[u, "\mathsf{gen}"] \arrow[r, "\Delta"] & A \times A \arrow[u, "\mathsf{id}\times\mathsf{gen}"']
\end{tikzcd}$$

$$\mathsf{gen} = [\![(\Delta)]\!]$$

Coinductive Definition

Stream Merge

$$\begin{tikzcd} A^\omega \arrow[r, "\langle \mathsf{hd},\mathsf{tl} \rangle"] & A \times A^\omega \\ A^\omega \times A^\omega \arrow[u, "\mathsf{merge}"] \arrow[r, "g"] & A \times (A^\omega \times A^\omega) \arrow[u, "\mathsf{id} \times \mathsf{merge}"'] \end{tikzcd}$$

$$\mathsf{merge} \; = \; [\![(g)]\!]$$

and

$$g \; = \; \langle \mathsf{hd} \cdot \pi_1, \mathsf{s} \cdot (\mathsf{tl} \times \mathsf{id}) \rangle$$

Coinductive Proof

$$\boxed{\textsf{merge}\ (a^\omega, b^\omega) \ = \ (ab)^\omega}$$

i.e.

$$\textsf{merge} \cdot (\textsf{gen} \times \textsf{gen}) \ = \ \textsf{twist}$$

where

$$\begin{tikzcd}
A^\omega \arrow[r, "\langle \textsf{hd}, \textsf{tl} \rangle"] & A \times A^\omega \\
A \times A \arrow[u, "\textsf{twist}"] \arrow[r, "\langle \pi_1, \textsf{s} \rangle"] & A \times (A \times A) \arrow[u, "\textsf{id} \times \textsf{twist}"']
\end{tikzcd}$$

Coinductive Proof

$$
\begin{aligned}
& \mathsf{merge} \cdot (\mathsf{gen} \times \mathsf{gen}) = \mathsf{twist} \\
= & \quad \{ \text{ definition } \} \\
& [\![(\langle \mathsf{hd} \cdot \pi_1, \mathsf{s} \cdot (\mathsf{tl} \times \mathsf{id}) \rangle)]\!] \cdot (\mathsf{gen} \times \mathsf{gen}) = \langle \pi_1, \mathsf{s} \rangle \\
\Leftarrow & \quad \{ \text{ fusion } \} \\
& \langle \mathsf{hd} \cdot \pi_1, \mathsf{s} \cdot (\mathsf{tl} \times \mathsf{id}) \rangle \cdot (\mathsf{gen} \times \mathsf{gen}) = \mathsf{id} \times (\mathsf{gen} \times \mathsf{gen}) \cdot \langle \pi_1, \mathsf{s} \rangle \\
= & \quad \{ \times \text{ absorption and reflection } \} \\
& \langle \mathsf{hd} \cdot \mathsf{gen} \cdot \pi_1, \mathsf{s} \cdot ((\mathsf{tl} \cdot \mathsf{gen}) \times \mathsf{gen}) \rangle = \mathsf{id} \times (\mathsf{gen} \times \mathsf{gen}) \cdot \langle \pi_1, \mathsf{s} \rangle \\
= & \quad \{ \mathsf{tl} \cdot \mathsf{gen} = \mathsf{gen} \text{ and } \mathsf{hd} \cdot \mathsf{gen} = \mathsf{id} \} \\
& \langle \pi_1, \mathsf{s} \cdot (\mathsf{gen} \times \mathsf{gen}) \rangle = \mathsf{id} \times (\mathsf{gen} \times \mathsf{gen}) \cdot \langle \pi_1, \mathsf{s} \rangle
\end{aligned}
$$

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= & \qquad \{\ \times\ \text{absorption}\ \} \\
& \langle \pi_1, \mathsf{s} \cdot (\mathsf{gen} \times \mathsf{gen}) \rangle \;=\; \langle \pi_1, (\mathsf{gen} \times \mathsf{gen}) \cdot \mathsf{s} \rangle \\
= & \qquad \{\ \mathsf{s}\ \text{natural}\ \} \\
& \langle \pi_1, \mathsf{s} \cdot (\mathsf{gen} \times \mathsf{gen}) \rangle \;=\; \langle \pi_1, \mathsf{s} \cdot (\mathsf{gen} \times \mathsf{gen}) \rangle
\end{array}
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$$\begin{tikzcd}
U \arrow[d, "p"'] & R \subseteq U \times V \arrow[l, "\pi_1"'] \arrow[r, "\pi_2"] \arrow[d, "\rho"'] & V \arrow[d, "q"'] \\
\mathsf{T}\, U & \mathsf{T}\, R \arrow[l, "\mathsf{T}\ \pi_1"'] \arrow[r, "\mathsf{T}\ \pi_2"] & \mathsf{T}\, V
\end{tikzcd}$$

Application: Concurrent Processes

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- The latter, encoded as a positive monoid, acts as a source of genericity

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- The latter, encoded as a positive monoid, acts as a source of genericity
- Equational (pointfree) reasoning (*vs* explicit bisimulations)
- Laws and constraints are 'found' (rather than postulated)

Application: Software Components

functions $f : I \longrightarrow O$ $f \in O^I$

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Components $\equiv$ seeded concrete coalgebras for Set endofunctors

$$\mathsf{T}^{\mathsf{B}} \;=\; \mathsf{B}\,(\mathsf{Id} \times O)^I$$

where B is a strong monad, capturing a behavioural model, *e.g.*,

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where B is a strong monad, capturing a behavioural model, *e.g.*,

- partiality: $\mathsf{B} = \mathsf{Id} + \mathbf{1}$
- non determinism: $\mathsf{B} = \mathcal{P}$
- monoidal stamping: $\mathsf{B} = \mathsf{Id} \times M$
- ‘metric’ non determinism: $\mathsf{B} = \mathsf{Bag}_M$

MSc Thesis Proposals

- Generic process calculi and development of parametric animators

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Logic & Formal Methods Group — the PURE Project