

On the 'A' that links the 'M's of Maths, Music and Maps

('Da capo')

J.N. Oliveira

Dept. Informática,
Universidade do Minho
Braga, Portugal

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Lab. de I&D de Computação Sonora e Musical
Dept. de Informática
FEUP, Porto

Prelude

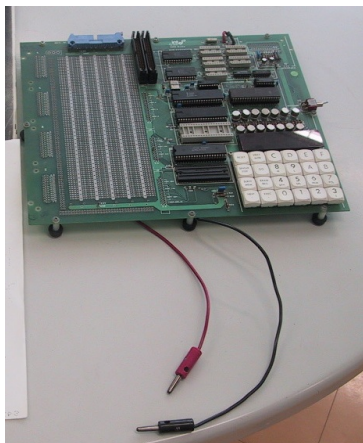
(dedicated to Raul Vidal)

Back to 1978/79

JUVEMINHO 79 — takes place in Braga in 1979 (*Parque de Exposições*).

U.Minho's **Electronics Laboratory** — recently created by Raul Vidal — prepares some demos for the exhibition.

Among these a multi-part **score interpreter** encoded in assembly code runs on an Intel SDK85, one of three bought by the Lab for the classroom.



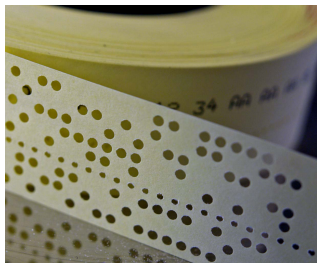
Back to 1978/79

Extra 2Kb memory extension has to be bought for the code to fit in.

No **assembler** available — code written directly in **hexadecimal** notation.

Program + 'scores' are saved in **punched tapes**.

Tapes are loaded from a **teletype** offered by CTT with no **documentation**, no **drivers** — that for SDK85 was built by trial-and-error on a breadboard.



Back to 1978/79

Written completely from scratch, the “synthesizer” sampled a 256 byte **sine wave** on a $\sqrt[12]{2}$ basis (everything calculated by the **Wang** machine in the D. Pedro V block, for SDK85 had no reals).

Part samples were produced in real-time, added together and sent to a **DA converter**, itself connected to an amplifier + speakers.

‘Scores’ included a few two part INVENTIONS (e.g. BWV 779) by J.S. Bach as well as the four-part Air of the SUITE BWV 1068.



Back to 1978/79

The code would need a little fix while at the exhibition, for notes of the same frequency in different parts (very) occasionally cancel each-other...

This was observed e.g. in the *Air* of BWV 1068, the superstar of the demo:



The image shows a musical score for the 'Air' from BWV 1068, featuring three staves: Treble Clef (Violin I), Treble Clef (Violin II), and Bass Clef (Cello/Double Bass). The score is in G major and 3/4 time. Two red boxes highlight specific measures where notes of the same frequency in different parts cancel each other out. A red musical note icon is also present in the bottom right corner of the score area.

Horror — no E's in the 1st and 2nd violins at one particular demo!

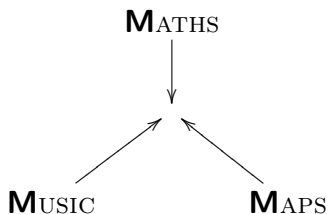
Fix thereafter ensured no two parts in phase opposition with each other.

Fugue

(35 years later...)

The “3M rule”

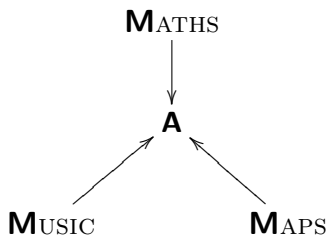
Someone has observed that *“those who like maths also enjoy music and maps”*¹:



What commonalities can be found between such a language system (Maths), an art (music) and a science (cartography)?

¹Comment by a student of David Naumann, Stevens Institute, New Jersey.

The “3M rule”



Maths is an *abstract* language (**A**)

Music is perhaps the most *abstract* (**A**) of all arts

Maps are geographical *abstractions* (**A**)

Abstraction

Quoting Jeff Kramer ²:

*“**Abstraction** is widely used in other disciplines such as **art and music**. For instance (...) Henri Matisse manages to clearly represent the **essence** of his subject, a naked woman, using only simple lines or cutouts. His representation **removes** all detail yet **conveys** much.”*



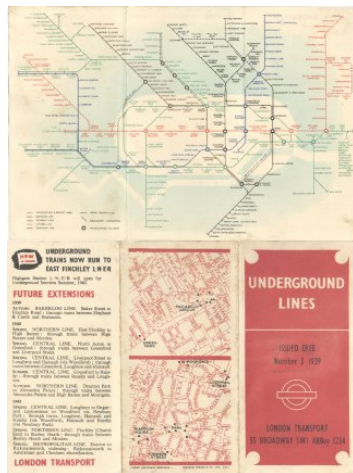
²*Is Abstraction the Key to Computing?*, Commun. ACM, 50:4, pages 37–42, April 2007.

Abstraction

*The famous “abstract”
map of the London
Underground (1939)*

BASE PRINCIPLE:

*“Eliminate characteristics
of the mapped object that
are **not relevant** to the
map’s **purpose**”*



Abstraction

Opus 118, no.2, by Johannes Brahms (1833-97):



Questions:

*What does this piece **mean**? Does it **describe** anything?
Does it **imitate** or recreate reality?*

Answer:

*It means **nothing** — it is **abstract**!*

Leonard Bernstein (1st *Young People's Concerts*, 18-Jan-1958):

"Music is never about anything: music just 'is'!"

Abstraction

Albert Camus (1913-60):

*Music is the **perfect expression** of an **ideal world** which is communicated to us through harmony. This world exists. Not at a level higher or lower than the real world, but parallel to it.*

***World of ideas?** Maybe. Or else **world of numbers**, as communicated to us by Harmony.”*

(Essay on Music, 1932)

Maths

Mathematics is the **universal** language of science. Why?

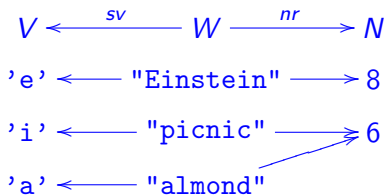
- The **abstract** language par excellence
- Safe (unambiguous) means to pass knowledge between generations
- Mathematical **proof** — the ideal way to provide verifiable **evidence**.

Abstraction: what is it, after all?

Our answer will be based on a concept of mathematics itself — that of a **function**.

Two functions

Number of letters (nr) and stress vowel (sv)



Notation (Leibniz): $6 = nr(\text{"almond"})$, $'a' = sv(\text{"almond"})$, etc

In general:

$$y = f(x)$$

Functional abstraction

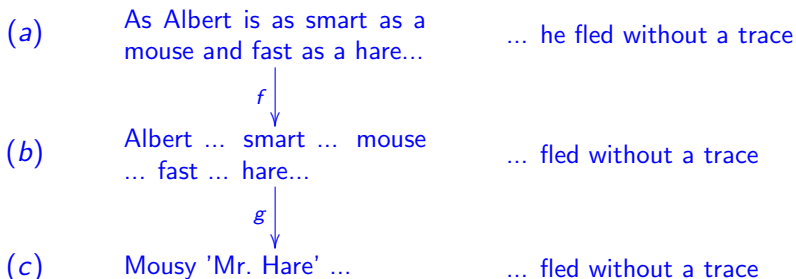
Given some function $A \xrightarrow{f} B$ such that, for all b in B , there is some a from A such that $b = f(a)$, for example

$$\begin{array}{l}
 W \xrightarrow{sv} V \\
 \text{"almond"} \longrightarrow \text{'a'} \quad (\text{etc}) \\
 \text{"Einstein"} \longrightarrow \text{'e'} \quad (\text{etc}) \\
 \text{"picnic"} \longrightarrow \text{'i'} \quad (\text{etc}) \\
 \text{"poll"} \longrightarrow \text{'o'} \quad (\text{etc}) \\
 \text{"future"} \longrightarrow \text{'u'} \quad (\text{etc})
 \end{array}$$

we say that domain B is **more abstract** than A and that f is a **witness** of such an abstraction.

In the example: **one** stress V owel abstracts **many** W ords.

Textual abstraction functions

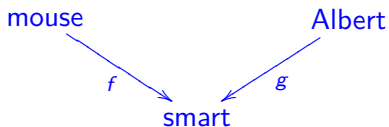


We see that by losing textual information, the text acquires a **metaphorical** dimension — e.g. nickname 'Hare', and so on.

“A la Chomsky”: functions f and g transform **deep structure** (a) into **surface structure** (c).

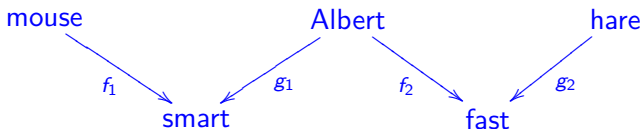
How two functions make a metaphor

Example:



where f and g are the **witnesses** of the metaphor.

In the example there are two juxtaposed metaphors:

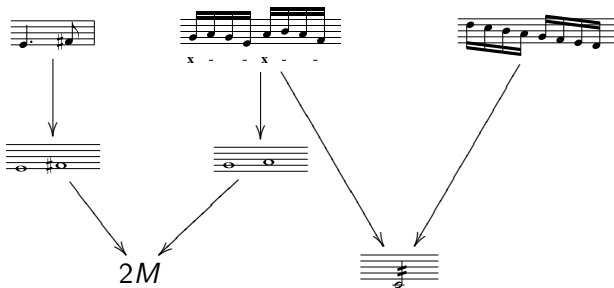


Metaphors in music

Listen to the music excerpt



which includes two simple metaphors, one **melodic** and the other **rhythmic**:

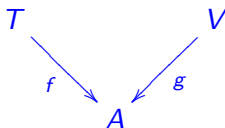


Metaphors as maths objects

A **metaphor** can be regarded as a “cospan” (aside) in which

- T (*tenor*) is the subject
- V is the *vehicle*
- A is the shared *attribute*.

(Cf. Richards' *Philosophy of Rhetoric*, 1936).



Summing up, a metaphor is a **binary relationship**

$$T(f \circ g)V \quad (1)$$

in which the attribute (A) is hidden.

Metaphors as maths objects

Brief explanation of the formula in the previous slide:

- R° denotes R in **passive voice**: $b R a$ thus means the same as $b R^\circ a$. Example:

Albert watches the hare

versus

the hare is watched by Albert

- Composition: $(f \cdot g)x = f(g x)$, for instance

$f p =$ the number of letters of p

$g p =$ the stressed syllable of p

Thus:

$(f \cdot g)p =$ the number of letters of the stressed syllable of p

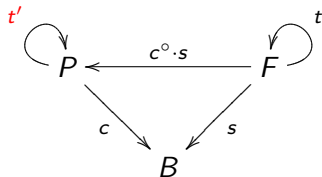
Putting the **vehicle** in motion

'No jobs for the boys' in metaphorical form, quoting Eça de Queirós (1845-1900):³

*“Os políticos e as fraldas
devem trocar-se
frequentemente e pela mesma
razão”*

*“Politicians and diapers
should be changed often and
for the same reason”*

Metaphor:

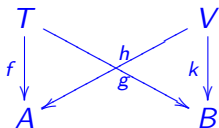


Axiom: $s(t x) = \text{False}$ —
inducing *change* t' in P , etc
etc.

³Abbreviations: P = politician (tenor); F = nappy (vehicle); c = *corrupted* ; s = *dirty* ; t = *change*.

Metaphors and ambiguity

In the presence of more than one shared attribute, e.g.



attribute omission leads to **ambiguity**.

Either **context** suggests the attribute or the text becomes **open** and may acquire a **poetic** dimension.

Metaphors often “close” inter-textually — see next slide.

Metaphors in music

A well-known anecdote (Vienna, 1820s):



Vaterländischer Künstlerverein (by 51 composers) on a waltz by Anton Diabelli (1781-1858).

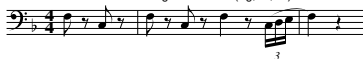
Var.XXII of contribution 51

A. Diabelli (1781-1858)



W.A. Mozart (1756-1791)

Don Giov.: Notte e giorno faticar (Fg,Vc,Ba)



176

Allegro molto .

Var: 22 .

Alla (Notte e giorno faticar) di Mozart.



Metaphors in music

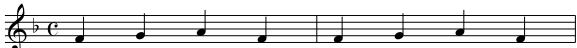
Funeral march by Gustav Mahler (1860-1911): metaphor with **tenor**

Symphonie nr. 1.iii 

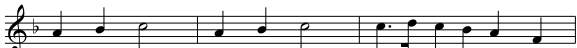
pp


whose **vehicle** is the popular:




Fre - - re Jac - - ques, Fre - - re Jac - - ques,



dor - mez vous? Dor - mez vous? Son-nez les ma-ti - nes!



Son-nez les ma-ti - nes! Din - dan - don. Din - dan - don. 

(More abstract) metaphors in music

3rd mov. of Brahms (1833-97) 1st symphony:



Sentence alone is metaphoric by itself, why?

- Too obvious: two halves share the same **rhythm**
- Less obvious: 2nd half (**tenor**) is an **inversion** of 1st half (**vehicle**).

Music can be very 'metaphoric' in this (rather **formal**) sense.

Chiasmus

José Saramago (1922-2010):

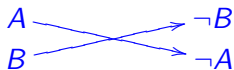
Deus está em mim,



ou em mim não está Deus

António Vieira (1608-97):

Os homens (tinham) a razão sem uso



os peixes o uso sem a razão

(shaped like the letter χ , cf. root $\chi\alpha\sigma\mu\acute{o}\varsigma$ from the Greek)

Chiasmus in music

The sequence $A B \neg B \neg A$ of terms that cross each other in the *parallel or double antithesis* of a **chiasmus** becomes **parallel de facto** in music, thanks to **polyphony**:



Antithesis by **retrograde** motion (cf. *algebraic* sequence inversion) exhibiting properties such as e.g.

$$\neg\neg A = A$$

$$\neg(A B) = (\neg B) (\neg A)$$

Chiasmus in music

Abundant device in e.g. Baroque music, cf. *Canon a 2 super thema regium* from BWV 1079 by J.S. Bach (1685-1750):

A =



B =



Musical Offering BWV 1079

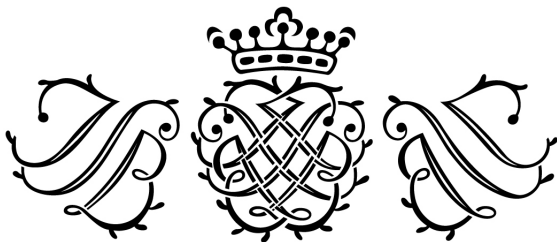
Canon a 2

(nr.1 of Canones diversi super thema regium, BWV 1079)

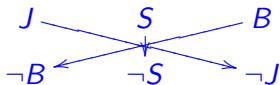
J.S. Bach (1685-1750)

The image displays the musical score for 'Canon a 2' from the 'Canones diversi super thema regium' (BWV 1079) by J.S. Bach. The score is written for two voices (treble and bass clefs) in B-flat major (two flats) and common time (C). It consists of five systems of two staves each. The notation includes various rhythmic values such as quarter, eighth, and sixteenth notes, as well as rests and accidentals. A red square icon with a musical note symbol is located at the bottom right of the score area.

Still about “chiasmus”



The stamp of J.S. Bach (1685-1750) in which the initials 'J', 'S', 'B' overlap each other once “mirrored” (ie. inverted), cf.



Algebraicity


Apparently a requirement “wilfully” claimed by Saussure in his writings (ELG, p. 236): ⁴

“l’expression simple sera algébrique ou ne sera pas”

In what measure can universal **algebra** help?

Music particularly interesting in this respect, for its inherent **algebraicity**:

- Music event level forms a very simple **algebra**: that of **sequences** of pairs (*pitch, duration*).
- Transformations across musical metaphor witnesses easy to express and reason about (many can be regarded as **linear transforms**).

⁴S. Bouquet, *Ontologie et Épistémologie de la Linguistique dans les Textes Originaux de Ferdinand de Saussure*, U. Paris X, 2008, vol. XVIII, no.3. 

Algebraicity

Take **augmentation**, for instance:

$$k * [] = []$$

$$k * [(p, d)] = [(p, k d)]$$

$$k * (m n) = (k * m) (k * n)$$

Transposition:

$$[] + i = []$$

$$[(p, d)] + i = [(p + i, d)]$$

$$(m n) + i = (m + i) (m + i)$$

Retrograde:

$$\neg[] = []$$

$$\neg[(p, d)] = [(p, d)]$$

$$\neg(m n) = (\neg n) (\neg m)$$

etc

Algebraicity

In general, can one “measure” algebraically the **expressive richness** of a metaphor $f^\circ \cdot g$?

Algebra of functions: $f \leq g$ measuring loss of information — f **abstracts** more than g ($\equiv f$ is less **injective** than g).

Distance between tenor and vehicle of metaphor $f^\circ \cdot g$ measurable by **complements** $\neg f$ and $\neg g$ (\equiv what is **not** common), where $\neg f$ satisfies the universal property

$$id \leq f \triangle k \equiv \neg f \leq k$$

where id is the identity and $(f \triangle g) x = (f x, g x)$.

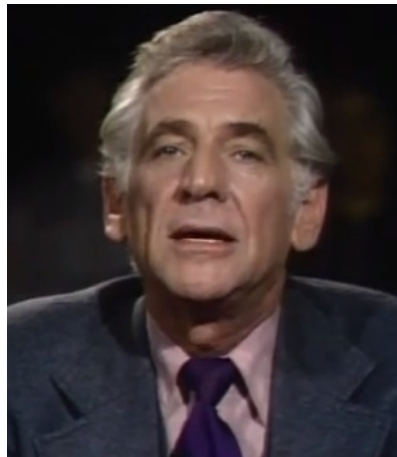
For example, small complements will correspond to *poor* metaphors, etc.

The Unanswered Question (Bernstein at Harvard)

*“He immersed himself in Chomskyan **linguistics** (...) so that he could then apply the principles of **linguistics to music** — thereby creating a brand new field of study.*

*Ambitious? Oh, yes!
Was he in over his head? Completely!”*

(Jamie Bernstein).



The Unanswered Question (Bernstein at Harvard)

For Leonard Bernstein (1919-90), music is

- inherently **transformational**
- the **most metaphorical** of all means of artistic expression.

In fact:

- musical processing functions are easy to identify (cf. **algebra** of sequences);
- metaphors in music are **inherently abstract** — they close up literally on the musical text alone.

Expressive efficacy proportional to the **'metaphorical engineering'** of the composer — have a look at our last example (next slides).

Creative process

The example leads us back to Chomsky's (hidden) **deep structures** (prosaic, vulgar) as opposed to the (visible, audible) **surface structures** (poetic, beautiful).

In a sense, isn't this **deep-to-surface** transformation an essential part of the **creative** process itself?

Sketch books of L. van Beethoven (1770-1827) — 7000 manuscript folia (many on the internet⁵) ready for this kind of study.

Deep structure starting point⁶: a vulgar, 'Mannheim-like' theme (see next slide).

⁵See *Beethoven sketches in the Digital Archives* available from <http://www.beethoven-haus-bonn.de>.

⁶Cf. B. Cooper, *Beethoven and the Creative Process*, Clarendon Press, 1990.

Creative process

Deep structure

SV236, fol.90v

erster Theil in B



From this, sonata opus 31-no.2 eventually emerged:

Op. 31. N^o2.

Largo. Allegro. Adagio.

mp p cresc. sf

5



Sketches unveil the metaphor

The image displays three musical sketches. The top-left sketch is in treble clef, starting with a 'Largo' tempo and transitioning to 'Allegro'. The top-right sketch is in bass clef, starting at '[m.20]' and transitioning to '[8.a.]'. Two arrows point from these sketches to a central sketch of the 'Mannheim rocket' theme in treble clef, which consists of a sequence of eighth notes: C4, D4, E4, F4, G4, A4, B4, C5, B4, A4, G4, F4, E4, D4, C4.

“Mannheim rocket” theme in the sketches is the hidden (common) attribute of the whole movement’s metaphor.

NB: see chap. 12 of *Cooper, op.cit.*, for a detailed study of the sketches of this movement of the sonata.

“Metaphors We Live By”

In their classic textbook, George Lakoff and Mark Johnson show how metaphorical “normal language” is in everyday life, eg.:

- ARGUMENT IS WAR (read: $Argument \xleftarrow{f \circ g} War$ for some witnessing f, g)
- TIME IS MONEY (read: $Time \xleftarrow{f \circ g} Money$ for some other witnesses f, g)

My question is:

*Since our brain structures concepts and knowledge around so many “down-to-earth” metaphors, how do such metaphors acquire the **aestetical** drive which lifts us to Camus’ (parallel) **ideal world**?*

Such is the magic of a **creative mind**.

Towards the Semantics of Music

Mihailo Antovic⁷ (Univ. Nis, Serbia):

*Music is an **abstraction**, and the only way to approach it is to **metaphorise** — i.e. map the **concrete** onto the **abstract**, be it through waterfalls, bamboos or dots on a vertical line.*

*(...) conceptual metaphor theory, in its search of the conceptualization of music, provides the most **solid grounds** for a true ‘**musico-semantics**’.*

⁷Antovic, M. (2009). Towards a Semantics of Music - the 20th Century, Language and History, 52(1): 119-129.

Afterthought

Two flavours in (applied) **linguistics**,

- **generative** (grammars, parsing)
- **cognitive** (“metaphors we live by” ...)

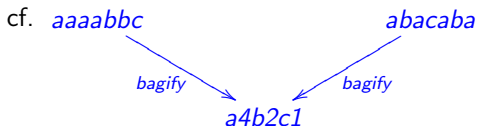
Parallel in software science:

- “Hylomorphisms” with pattern $f \cdot g^\circ$, eg. context-free compilers:

$$\text{compiler} = \text{code_generator} \cdot \text{pretty_printer}^\circ$$

- “Metaphorisms” with pattern $f^\circ \cdot g$, eg. sorting:

$$\text{sort} = \text{is_ordered} \cdot (\text{bagify}^\circ \cdot \text{bagify})$$



Epilogue

Humanities versus Science

- The *Big Divide*: “Art” or “Science” ... — a historical error.
- Nefarious disjunction since the era of specialization.
- Man of Hellenism and of the Renaissance lost.
- Some *counter-examples* in Portugal: João de Freitas Branco (1922-89), Jorge de Sena (1919-78), Rómulo de Carvalho (1906-97) ...
- Sociology of mathematics — the real problem?
- Confluence requires change of attitude on both sides, with much work ahead.

Music well positioned to “bridge the gap” ...

- Experience in **Computer-aided Musicology** course at Minho.

Epilogue

I wish a long and fruitful life to the newly reborn

*Lab. de I&D de Computação Sonora e
Musical @ FEUP*

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