Tool Support for Learning Büchi Automata and Linear Temporal Logic

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Background

- **Büchi automata** and **linear temporal logic** are two fundamental components of *model checking*, in particular, the automata-theoretic approach:
  - The (finite-state) system is modeled as a Büchi automaton A.
  - A desired behavioral property of the system is given by a linear temporal formula f.
  - Let B_f ($B_{\sim f}$) denote a Büchi automaton equivalent to f ($\sim f$).
  - The model checking problem is essentially asking whether

$$L(A) \subseteq L(B_f) \text{ or equivalently } L(A) \cap L(B_{\sim f}) = \emptyset \text{ .}$$

- The well-used model checker SPIN, for example, adopted the automata-theoretic approach.

Motivation

- Model checking has proven to be very useful and the number of courses covering related topics appears to be increasing.
- Understanding the correspondence between Büchi automata and linear temporal logic is not easy.
- A graphical interactive tool may be helpful for the learner (and the teacher).
- Tools exist for learning *classic* automata and formal languages, e.g., JFLAP (which inspired our tool GOAL and provided some of its basic building blocks).

Büchi Automata

- Büchi automata (BA) are a variant of omega-automata, which are finite automata operating on *infinite* words.
- A Büchi automaton is given, as in finite automata, by a 5-tuple $(\Sigma, Q, \delta, Q_0, F)$, where $F \subseteq Q$ is the set of accepting states.
- An infinite word $\alpha \in \Sigma^\omega$ is accepted by a Büchi automaton *B* if there exists a run ρ of *B* on α satisfying the condition: $Inf(\rho) \cap F \neq \emptyset$

  where $Inf(\rho)$ denotes the set of states occurring infinitely many times in ρ.

Generalized Büchi Automata

- A generalized Büchi automaton (GBA) is like a BA but with $F \subseteq 2^Q$, i.e., $F = \{F_1,\ldots,F_k\}$ where $F_i \subseteq Q$.

- A word $\alpha \in \Sigma^\omega$ is accepted by a generalized Büchi automaton $\mathcal{B}$ if there exists a run ρ of $\mathcal{B}$ on α satisfying the condition:

$$\forall F_i \in F: \; Inf(\rho) \cap F_i \neq \emptyset$$

About the Alphabet

- To link Büchi automata to temporal formulae, we will consider automata with an alphabet like:
  - {p,~p}
  - {pq,p~q,~pq,~p~q}

Propositional Linear Temporal Logic (PTL)

- A subset of linear temporal logic (LTL).
- PTL formulae are interpreted over an infinite sequence of states, which can be seen as an infinite word over a suitable alphabet like {p,~p} or {pq,p~q,~pq,~p~q}.
- *Every PTL formula is equivalent to some Büchi automaton, but not vice versa.*

Note: Quantified PTL (QPTL) are as expressive as Büchi automata.

Temporal Operators in PTL

- Future temporal operators:
  - next: ( ) or X
  - eventually (sometime): <> or F
  - hence-forth (always): [ ] or G
  - wait-for (unless): *W*
  - until: *U*
- Past temporal operators:
  - previous: (-) or Y
  - before: (~) or Z
  - once: <-> or O
  - so-far: [-] or H
  - back-to: *B*
  - since: *S*

Semantics of Future Operators

Let π be an infinite sequence of states.

- $(\pi, i) \vDash (\,)f$ iff $(\pi, i+1) \vDash f$
- $(\pi, i) \vDash \Diamond f$ iff $(\pi, j) \vDash f$ for some $j \geq i$
- $(\pi, i) \vDash [\,]f$ iff $(\pi, j) \vDash f$ for all $j \geq i$
- $(\pi, i) \vDash f\,U\,g$ iff $(\pi, k) \vDash g$ for some $k \geq i$ and $(\pi, j) \vDash f$ for all j, $i \leq j < k$
- $(\pi, i) \vDash f\,W\,g$ iff $(\pi, i) \vDash [\,]f$ or $(\pi, i) \vDash f\,U\,g$

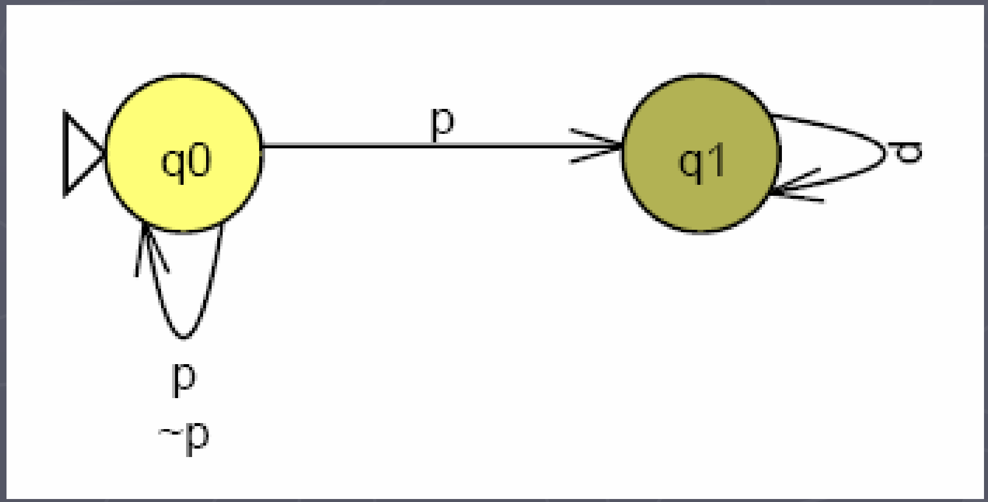
Semantics of Past Operators

- $(\pi, i) \vDash (-)f$ iff $i \geq 1$ and $(\pi, i\text{-}1) \vDash f$
- $(\pi, i) \vDash (\sim)f$ iff $i = 0$ or $(\pi, i\text{-}1) \vDash f$
- $(\pi, i) \vDash \langle - \rangle f$ iff $(\pi, j) \vDash f$ for some j, $0 \leq j \leq i$
- $(\pi, i) \vDash [-]f$ iff $(\pi, j) \vDash f$ for all j, $0 \leq j \leq i$
- $(\pi, i) \vDash f\,\mathcal{S}\,g$ iff $(\pi, k) \vDash g$ for some $k \leq i$,
  and $(\pi, j) \vDash f$ for all j, $k < j \leq i$
- $(\pi, i) \vDash f\,\mathcal{B}\,g$ iff $(\pi, i) \vDash [-]f$ or $(\pi, i) \vDash f\,\mathcal{S}\,g$

Example 1: <>[]p

- Meaning: p always holds after some time
- Satisfying models:
  - $(p)^{\omega}$, i.e., ppp…
  - $p{\sim}p{\sim}pp{\sim}p(p)^{\omega}$
- Unsatisfying models:
  - $p{\sim}p{\sim}pp({\sim}pp)^{\omega}$

<>[]p as a Büchi Automaton

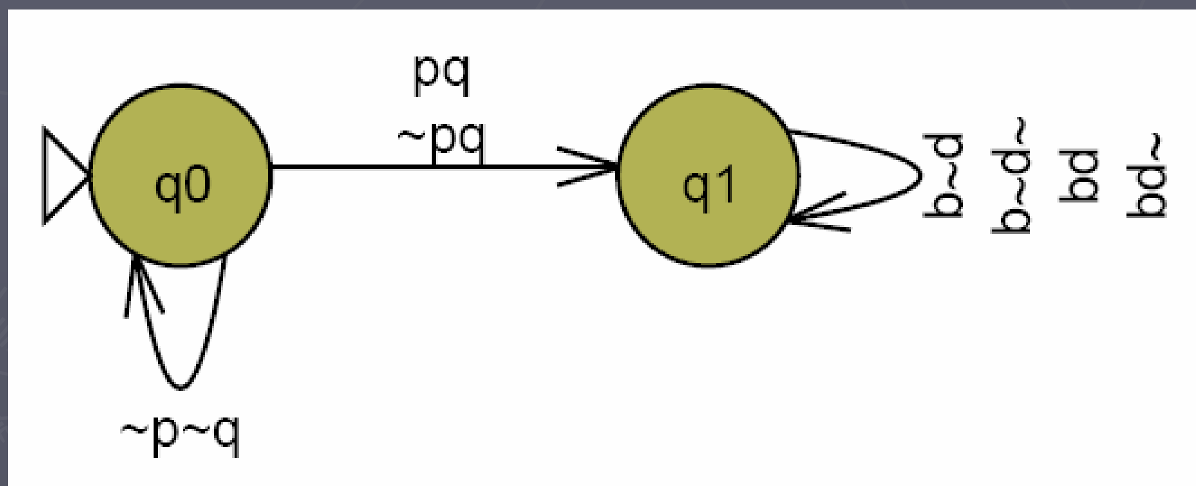


F = {q1}

Example 2: [](p --> <->q)

- Meaning: Every p is preceded by q.
- Satisfying models:
  - $(\sim p \sim q)^{\omega}$
  - $(\sim p \sim q)(\sim pq)\ (\sim p \sim q)\ (p \sim q)^{\omega}$
- Unsatisfying models:
  - $(\sim p \sim q)(p \sim q)\ldots$

[](p --> <->q) as a Büchi Automaton

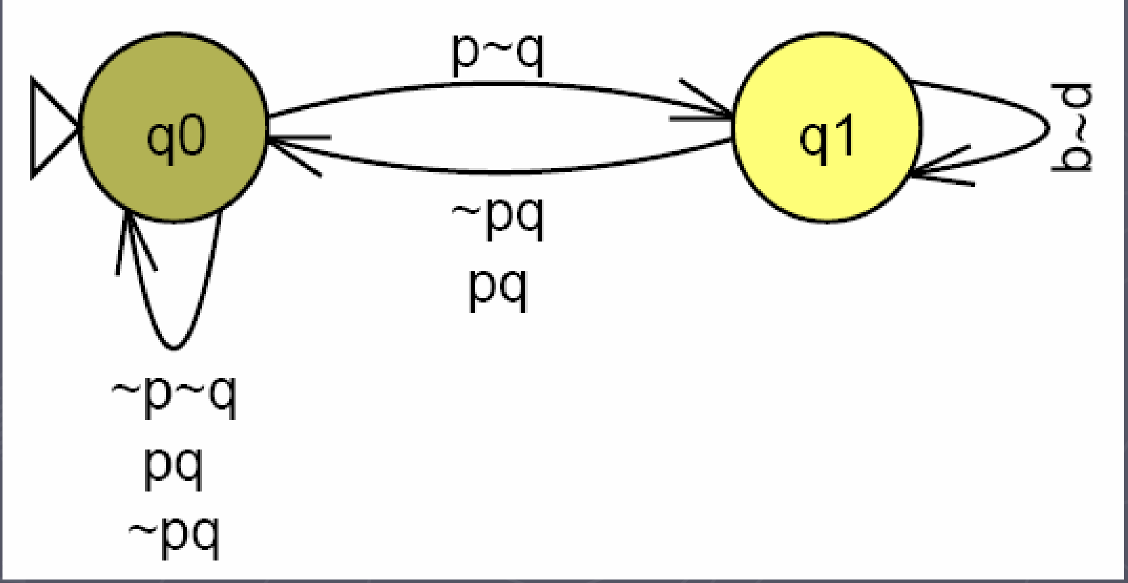


F = {q0,q1}

Example 3: [](p --> p *U* q)

- Meaning: Once p becomes true, it will remain true continuously until q becomes true, and q does become true.
- Satisfying models:
  - $(\sim p \sim q)^{\omega}$
  - $(\sim p \sim q)(p \sim q)(p \sim q)(p \sim q)(\sim pq)(\sim p \sim q)^{\omega}$
- Unsatisfying models:
  - (~p~q)(p~q)(~p~q)…

[](p --> p *U* q) as a Büchi Automaton



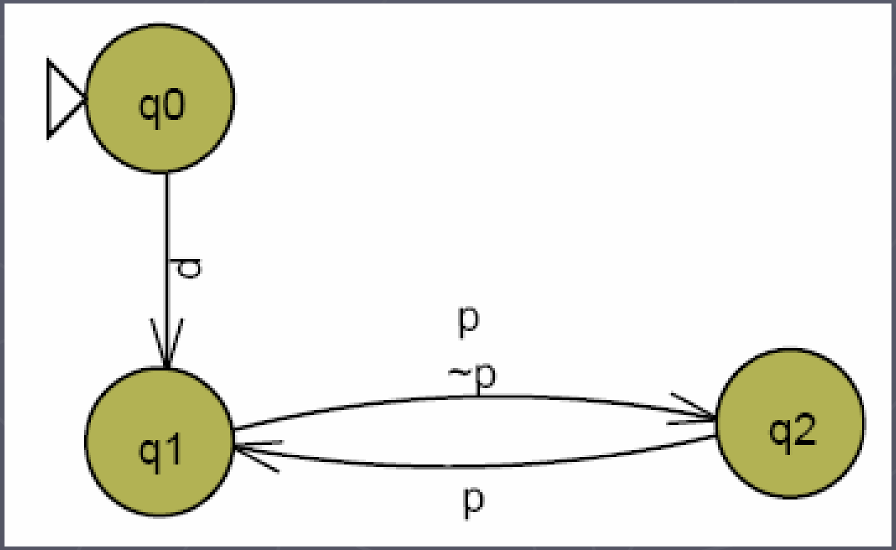
F = {q0}

Example 4: "Even p"

- This is NOT a PTL formula!
- Meaning: p holds in very even state.

  (Note: the states of a sequence are numbered 0,1,2,3,…)
- Satisfying models:
  - $(p)^\omega$
  - $(p\sim p)^\omega$
  - $p\sim pp\sim p(pp)^\omega$
- Unsatisfying models:
  - $p\sim ppp\sim p(pp)^\omega$

“Even p” as a Büchi Automaton



F = {q0,q1,q2}

Main Features of GOAL

- Drawing and Running Büchi Automata
- PTL Formulae to BA Translation
- Boolean Operations on BA
  - Union
  - Intersection
  - Complement
- Tests on BA
  - Emptiness
  - Containment (language containment)
  - Equivalence (language equivalence)
- Repositories of pre-drawn BA.

Test Running a BA

- To get an intuitive understanding of what language is being defined by the BA.
- Input format
  - Input string: ppp~pp(~pp)$^{\omega}$
    Real format: (p)(p)(p)(~p)(p){(~p)(p)}
  - Input string: (~pq) ((~pq) (~p~q) (~p~q))$^{\omega}$
    Real format: (~pq) **{**(~pq) (~p~q) (~p~q)**}**

Demo Script

- Draw a BA, intended for <>[]p.
- Check if it is correct, by comparing with a machine-translated one.
- Try to specify “Even p” in PTL.
- See why it fails.
- Perhaps more …

The Future of GOAL

GOAL is constantly being improved; possible future extensions include:

- Integration with LTL model checkers
  - For example, export automata as Promela code for SPIN
- QPTL, PSL, S1S, etc. to Büchi automata (and vice versa)
- Minimization of Büchi automata
- Transformation to and from other variants of ω-automata
- Even better editing environment
  - Faster local updates in large graph layouts