

# Pointfree Alloy: the other side of the moon

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# Model driven engineering

- **MEDEA** project — *High Assurance MDE using Alloy*
- **MDE** is a clumsy area of work, full of approaches, acronyms, notations.
- **UML** has taken the lead in *unifying* such notations, but it is too **informal** to be accepted as a reference approach.
- Model-oriented formal methods (**VDM**, **Z**) solve this informality problem at a high-cost: people find it hard to understand models written in maths (cf. maths illiteracy if not mathphobic behaviour).
- **Alloy** [4] offers a good compromise — it is formal in a light-weight manner.

# Inspiration

- **BBI** project [5]: **Alloy** re-engineering of a well-tested, very well written non-trivial prototype in **Haskell** of a real-estate trading system similar to the stocks market (65 pages in lhs format) unveiled 4 bugs (2 invariant violations + 2 weak pre-conditions)
- Alloy and Haskell complementary to each other

# Alloy

## What **Alloy** offers

- A unified approach to **modeling** based on the notion of a **relation** — “**everything is a relation**” in Alloy.
- A minimal syntax (centered upon relational composition) with an object-oriented flavour which captures much of what otherwise would demand for **UML+OCL**.
- A **pointfree** subset.
- A model-checker for model assertions (counter-examples within scope).

# Alloy

What **Alloy** **does not** offer

- Complete calculus for deduction (proof theory)
- Strong type checking
- Dynamic semantics modeling features

Opportunities

- Enrich the standard Alloy *modus operandi* with relational algebra calculational proofs
- Connect the tool to a theorem prover, eg. **Prover9** as suggested in [3]
- Design an Alloy-centric tool-chain for high assurance model-oriented design

Thus the **MEDEA** project (submitted).

# Relational composition

- The swiss army knife of Alloy
- It subsumes function application and “field selection”
- Encourages a navigational (point-free) style based on pattern  $x.(R.S)$ .
- Example:

$Person = \{(P1), (P2), (P3), (P4)\}$

$parent = \{(P1, P2), (P1, P3), (P2, P4)\}$

$me = \{(P1)\}$

$me.parent = \{(P2), (P3)\}$

$me.parent.parent = \{(P4)\}$

$Person.parent = \{(P2), (P3), (P4)\}$

## When “everything is a relation”

- Sets are relations of arity 1, eg.  
 $Person = \{(P1), (P2), (P3), (P4)\}$
- Scalars are relations with size 1, eg.  $me = \{(P1)\}$
- Relations are first order, but there are multi-ary relations.
- However, **Alloy** relations are not  $n$ -ary in the usual sense: instead of thinking of  $R \in 2^{A \times B \times C}$  as a set of triples (there is no such thing as *tupling* in Alloy), think of  $R$  in terms of *currying*:

$$R \in (B \rightarrow C)^A$$

(More about this later.)

## Kleene algebra flavour

Basic operators:

.	<i>composition</i>
+	<i>union</i>
$\wedge$	<i>transitive closure</i>
*	<i>transitive-reflexive closure</i>

(There is no explicit recursion in **Alloy**.) Other relational operators:

$\sim$	<i>converse</i>
++	<i>override</i>
$\&$	<i>intersection</i>
-	<i>difference</i>
->	<i>cartesian product</i>
<:	<i>domain restriction</i>
:>	<i>range restriction</i>



## Relational thinking

- As a rule, thinking in terms of poinfree relations (this includes **functions**, of course) pays the effort: the concepts and the reasoning become simpler.
- This includes **relational data** structuring, which is far more interesting than what can be found in SQL and relational databases.

### Example — list processing

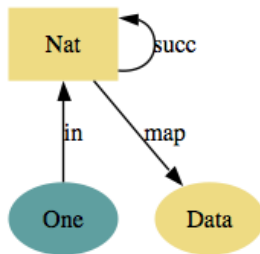
- **Lists** are traditionally viewed as recursive (linear) data structures.
- There are no lists in Alloy — they have to be modeled by **simple** relations (vulg. partial functions) between indices and elements.

# Lists as relations in Alloy

```
sig List {  
    map : Nat -> lone Data  
}
```

```
sig Nat {  
    succ: one Nat  
}
```

```
one sig One in Nat {}
```



Multiplicities: **lone** (one or less), **one** (exactly one)

# Relational data structuring

Some correspondences:

<b>list</b> $l$	<b>relation</b> $L$
sorted	monotonic
no-duplicates	injective
$map\ f\ l$	$f \cdot L$
$zip\ l_1\ l_2$	$\langle L_1, L_2 \rangle$
$[1, \dots]$	$id$

where

- $id$  is the identity (equivalence) relation (also a function)
- the “fork” (also known as “split”) combinator  $\langle -, - \rangle$  is such that  $(x, y) \langle L_1, L_2 \rangle z$  means the same as  $xL_1z \wedge yL_2z$

# Haskell versus Alloy

Pointwise Haskell:

```
findIndices      :: (a -> Bool) -> [a] -> [Int]
findIndices p xs = [ i | (x,i) <- zip xs [0..], p x ]
```

Pointfree (PF):

$$\text{findIndices } p \ L \triangleq \pi_2 \cdot (\Phi_p \times id) \cdot \langle L, id \rangle \quad (1)$$

where

- $\pi_2$  is the right projection of a pair
- $L \times R = \langle L \cdot \pi_1, R \cdot \pi_2 \rangle$
- $\Phi_p \subseteq id$  is the coreflexive relation (partial identity) which models predicate  $p$  (or a set)

# Haskell versus Alloy

- What about Alloy? It has no pairs, therefore no forks  $\langle L, R \rangle \dots$
- Don't worry — Alloy is relational and we can play with the relational calculus:

$$\begin{aligned} & \pi_2 \cdot (\Phi_p \times id) \cdot \langle L, id \rangle \\ = & \quad \{ \text{×-absorption} \} \\ & \pi_2 \cdot \langle \Phi_p \cdot L, id \rangle \\ = & \quad \{ \text{×-cancelation} \} \\ & \delta(\Phi_p \cdot L) \end{aligned}$$

where  $\delta$  is the **domain** operator:  $\delta R = R^\circ \cdot R \cap id$ , for  $R^\circ$  the converse of  $R$ .

# Haskell versus Alloy

Two ways of writing  $\delta(\Phi_p \cdot L)$  in Alloy, one pointwise

```
fun findIndices[s:set Data,l:List]: set Nat {  
    {i: Nat | some x:s | x in i.(l.map)}  
}
```

and the other pointfree,

```
fun findIndices[s:set Data,l:List]: set Nat {  
    dom[l.map :> s]  
}
```

the latter corresponding to what we've calculated.

## Beyond model-checking: proofs by calculation

Suppose the following property

$$(findIndices\ p) \cdot r^{\star} = findIndices\ (p \cdot r) \quad (2)$$

is asserted in (pointwise) Alloy:

```
assert FT {
  all l,l':List, p: set Data, r: Data -> one Data |
    l'.map = l.map.r =>
      findIndices[p,l'] = findIndices[r.p,l]
}
```

**NB:** the following version of (2) explains the encoding above:

$$r^{\star} \subseteq (findIndices\ p)^{\circ} \cdot findIndices\ (p \cdot r)$$

whereby, going pointwise, we get

$$l' = r^{\star} \ l \Rightarrow findIndices\ p \ l' = findIndices(p \cdot r) \ l$$

## Beyond model-checking: proofs by calculation

- Suppose the **Alloy** model checker does not yield any counter-examples for this property, for increasing bounds.
- How can we be sure of its validity?
  - Free theorems — the given assertion is a corollary of the **free-theorem** [6] of *findIndices*, thus there is nothing to prove (model checking could altogether be avoided!)
  - Wishing to prove the assertion anyway, one calculates:



# Trivial proof

$$(findIndices\ p) \cdot r^* = findIndices\ (p \cdot r)$$

$$\Leftrightarrow \quad \{ \text{list to relation transform} \}$$

$$\delta(\Phi_p \cdot (r \cdot L)) = \delta(\Phi_{p \cdot r} \cdot L)$$

$$\Leftrightarrow \quad \{ \text{property } \Phi_{f \cdot g} = \delta(\Phi_f \cdot g) \}$$

$$\delta(\Phi_p \cdot (r \cdot L)) = \delta(\delta(\Phi_p \cdot r) \cdot L)$$

$$\Leftrightarrow \quad \{ \text{domain of composition} \}$$

$$\delta(\Phi_p \cdot (r \cdot L)) = \delta((\Phi_p \cdot r) \cdot L)$$

$$\Leftrightarrow \quad \{ \text{associativity} \}$$

$$\text{TRUE}$$

# Realistic example — Verified FSystem (VFS)

**VERIFYING INTEL'S FLASH FILE SYSTEM CORE**  
Miguel Ferreira and Samuel Silva  
University of Minho  
{pg10961,pg11034}@alunos.uminho.pt

*Deep Space lost contact with Spirit on 21 Jan 2004, just 17 days after landing.*

*Initially thought to be due to thunderstorm over Australia.*

*Spirit transmitted an empty message and missed another communication session.*

*After two days controllers were surprised to receive a relay of data from Spirit.*

*Spirit didn't perform any scientific activities for 10 days.*





*This was the most serious anomaly in four-year mission.*

*Fault caused by Spirit's FLASH memory subsystem*

**Why formal methods?**  
Software bugs cost millions of dollars.

**What we can do?**  
Build abstract models (VDM).  
Gain confidence on models (Alloy).  
Proof correctness (HOL & PF-Transform).

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Thanks to Peter Gorm Larsen for VDMTools support.



# VFS in Alloy (simplified)

The system:

```
sig System {  
  fileStore: Path -> lone File,  
  table: FileHandle -> lone OpenFileInfo  
}
```

Paths:

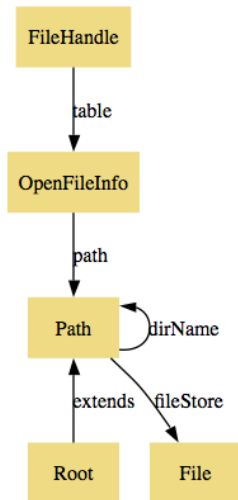
```
sig Path {  
  dirName: one Path  
}
```

The root is a path:

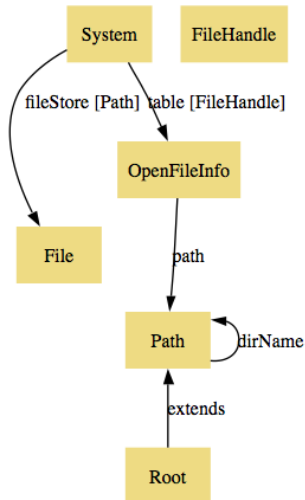
```
one sig Root extends Path {  
}
```

# Alloy diagrams for FSystem

Simplified:



Complete:



# Binary relation semantics

Meaning of signatures:

```
sig Path {  
  dirName: one Path  
}
```

declares function  $Path \xrightarrow{\text{dirName}} Path$ .

```
sig System {  
  fileStore: Path -> lone File,  
}
```

declares simple relation  $System \times Path \xrightarrow{\text{fileStore}} File$ .

(NB: a relation  $S$  is **simple**, or *functional*, wherever its **image**  $S \cdot S^\circ$  is coreflexive. Using harpoon arrows  $\rightarrow$  for singling these out.)

## Binary relation semantics

- Since

$$(A \times B) \rightarrow C \cong (B \rightarrow C)^A$$

*fileStore* can be alternatively regarded as a function in  $(Path \rightarrow File)^{System}$ , that is, for  $s : System$ ,

$$Path \xrightarrow{(fileStore\ s)} File$$

- Thus the “navigation-styled” notation of **Alloy**:  $p.(s.fileStore)$  means the file accessible from path  $p$  in file system  $s$ .
- Similarly, line `table: FileHandle -> lone OpenFileInfo` in the model declares

$$FileHandle \xrightarrow{(table\ s)} OpenFileInfo$$

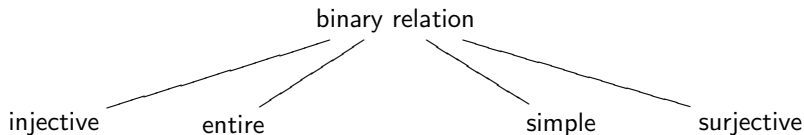
# Multiplicities in Alloy + taxonomy

A lone $\rightarrow$ B	A $\rightarrow$ some B	A $\rightarrow$ lone B	A some $\rightarrow$ B
injective	entire	simple	surjective
A lone $\rightarrow$ some B	A $\rightarrow$ one B	A some $\rightarrow$ lone B	
representation	function	abstraction	
A lone $\rightarrow$ one B		A some $\rightarrow$ one B	
injection		surjection	
A one $\rightarrow$ one B			
bijection			

(courtesy of Alcino Cunha, Alloy expert at Minho)

# The same — mathematically

## Topmost criteria:



## Definitions:

	<i>Reflexive</i>	<i>Coreflexive</i>
$\ker R$	entire $R$	injective $R$
$\text{img } R$	surjective $R$	simple $R$

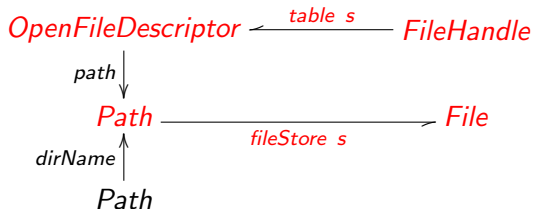
$$\ker R = R^\circ \cdot R$$

$$\text{img } R = R \cdot R^\circ$$



# From Alloy to relational diagrams

We draw



where

- *table s* and *fileStore s* are simple relations
- the other arrows depict functions

(Diagram in the **Rel** allegory [1] to be completed soon.)

# Model constraints

Referential integrity:

*Non-existing files cannot be opened:*

```
pred ri[s: System]{  
    all h: FileHandle, o: h.(s.table) |  
        some (o.path).(s.fileStore)  
}
```

Paths closure:

*Mother directories exist and are indeed directories:*

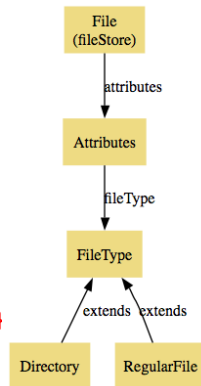
```
pred pc[s: System]{  
    all p: Path |  
        some p.(s.fileStore) =>  
            (some d: (p.dirName).(s.fileStore) |  
                d.fileType=Directory)  
}
```

## 2nd part of Alloy FSystem model

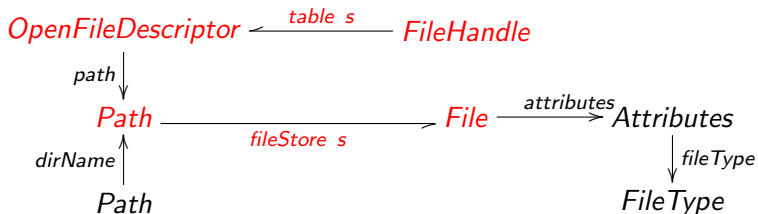
```
sig File {  
    attributes: one Attributes  
}
```

```
sig Attributes{  
    fileType: one FileType  
}
```

```
abstract sig FileType {}  
one sig RegularFile extends FileType {}  
one sig Directory extends FileType {}
```



## Updated binary relational diagram



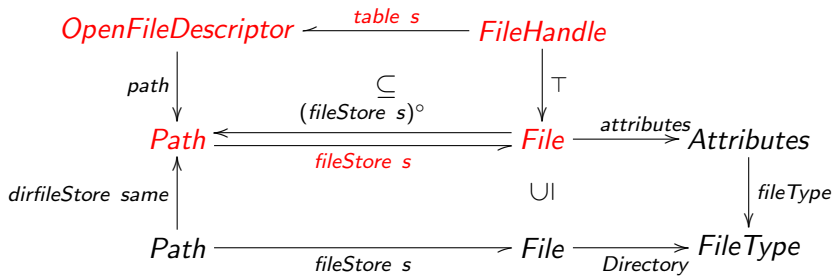
where

- *table s*, *fileStore s* are simple relations
- all the other arrows depict functions

Constraints: still missing

## Updating diagram with constraints

Complete diagram, where Directory is the “everywhere-*Directory*” constant function:



Constraints:

- Top rectangle is the PF-transform of *ri* (referential integrity)
- Bottom rectangle is the PF-transform of *pc* (path closure)

# PF-constraints in symbols

## Referential integrity:

$$ri(s) \triangleq path \cdot (table\ s) \subseteq (fileStore\ s)^\circ \cdot \top \quad (3)$$

which is equivalent to

$$ri(s) \triangleq \rho(path \cdot (table\ s)) \subseteq \delta(fileStore\ s)$$

where  $\rho R = \delta R^\circ$ . PF version (3) nicely encoded in **Alloy**

```
pred riPF[s: System]{  
    s.table.path in (FileHandle->File).~(s.fileStore)  
}
```

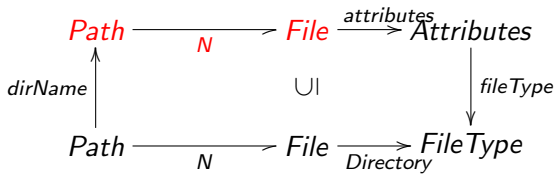
thanks to its emphasis on **composition**.

# PF-constraints in symbols

## Paths closure:

$$pc\ N \triangleq \underline{Directory} \cdot N \subseteq fileType \cdot attributes \cdot N \cdot dirName \quad (4)$$

recall lower part of diagram:



Again thanks to emphasis on **composition**, this is easily encoded in PF-Alloy:

```

pred pcPF[s: System]{
    s.fileStore.(File->Directory) in
        dirName.(s.fileStore).attributes.fileType
}
  
```

## PF-ESC by calculation

- Models with constraints put the burden on the designer to ensure that operations **type-check** (read this in **extended-mode**), that is, constraints are preserved across the models operations.
- Typical approach in MDE: **model-checking**
- Automatic **theorem proving** also considered in safety-critical systems
- However: convoluted pointwise formulæ often lead to failure.

How about doing these as “pen & paper” exercises?

- **PF-formulæ** are manageable, this is the difference.



## Example of PF-ESC by calculation

Consider the operation which removes file system objects, as modeled in Alloy:

```
pred delete[s',s: System, sp: set Path]{  
    s'.table = s.table  
    s'.fileStore = (univ-sp) <: s.fileStore  
}
```

that is,

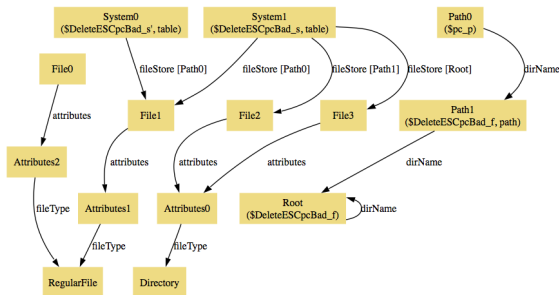
$$\textit{delete } sp \ (M, N) \triangleq (M, N \cdot \Phi_{(\neg sp)}) \quad (5)$$

where  $\Phi_{(\neg sp)}$  is the coreflexive associated to the complement of  $sp$ .

# Intuitive steps

Intuitively, *delete* will put at risk

- the *ri* constraint once we decide to delete file system objects which are open
- the *pc* constraint once we decide to delete directories with children.



(Model-checking in **Alloy** will easily spot these flaws, as checked above by a counter-example for the latter situation.)

## Intuitive steps

We have to guess a **pre-conditions** for *delete*. However,

- How can we be sure that such (guessed) pre-condition is *good enough*?
- The best way is to calculate the weakest pre-condition for each constraint to be maintained.
- In doing this, mind the following properties of relational algebra:

$$h \cdot R \subseteq S \Leftrightarrow R \subseteq h^\circ \cdot S \quad (6)$$

$$R \cdot \Phi = R \cap \top \cdot \Phi \quad (7)$$

$$f \cdot R \subseteq \top \cdot S \Leftrightarrow R \subseteq \top \cdot S \quad (8)$$

For improved readability, we introduce abbreviations

$ft := \text{fileType} \cdot \text{attributes}$  and  $d := \underline{\text{Directory}}$ , and **calculate**:

## Calculational steps

$pc(delete\ S\ (M, N))$

$\Leftrightarrow \quad \{ \text{(5) and (4)} \}$

$d \cdot (N \cdot \Phi_{(\not\in S)}) \subseteq ft \cdot (N \cdot \Phi_{(\not\in S)}) \cdot dirName$

$\Leftrightarrow \quad \{ \text{shunting (6)} \}$

$d \cdot N \cdot \Phi_{(\not\in S)} \cdot dirName^\circ \subseteq ft \cdot N \cdot \Phi_{(\not\in S)}$

$\Leftrightarrow \quad \{ \text{(7)} \}$

$d \cdot N \cdot \Phi_{(\not\in S)} \cdot dirName^\circ \subseteq ft \cdot N \cap \top \cdot \Phi_{(\not\in S)}$

$\Leftrightarrow \quad \{ \text{\(\cap\)-universal ; shunting} \}$

## Ensuring paths closure

$$\begin{cases} d \cdot N \cdot \Phi_{(\notin S)} \subseteq ft \cdot N \cdot dirName \\ d \cdot N \cdot \Phi_{(\notin S)} \subseteq \top \cdot \Phi_{(\notin S)} \cdot dirName \end{cases}$$

$$\Leftrightarrow \quad \{ \text{ } \top \text{ absorbs } d \text{ (8)} \}$$

$$\begin{cases} \underbrace{d \cdot N \cdot \Phi_{(\notin S)} \subseteq ft \cdot N \cdot dirName}_{\text{weaker than } pc(N)} \\ \underbrace{N \cdot \Phi_{(\notin S)} \subseteq \top \cdot \Phi_{(\notin S)} \cdot dirName}_{wp} \end{cases}$$

Back to points, *wp* is:

$$\langle \forall q : q \in dom N \wedge q \notin S : dirName q \notin S \rangle$$

$$\Leftrightarrow \quad \{ \text{ predicate logic } \}$$

$$\langle \forall q : q \in dom N \wedge (dirName q) \in S : q \in S \rangle$$

## Ensuring paths closure

In words:

*if parent directory of existing path  $q$  is marked for deletion then so must be  $q$ .*

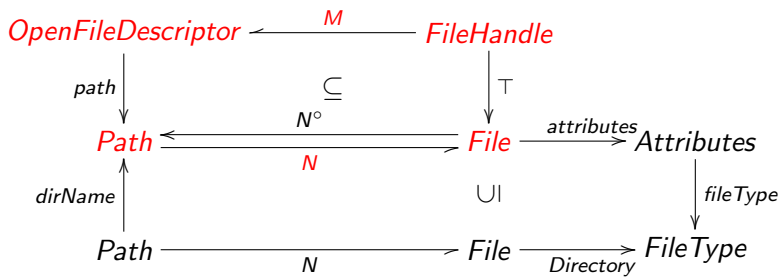
Translating calculated weakest precondition back to Alloy:

```
pred pre_delete[s: System, sp: set Path]{
    all q: Path |
        some q.(s.fileStore) &&
            q.dirName in sp => q in sp
}
```

## Back to the diagram

PF-encoding of model constraints in terms of relational composition has at least the following advantages:

- it makes **calculations** easier (rich algebra of  $R \cdot S$ )
- it makes it possible to **draw** constraints as rectangles in diagrams, recall



- it enables the “navigation-styled” notation of Alloy

# Constraint bestiary

- Experience in formal modeling tells that designs are **repetitive** in the sense that they instantiate (**generic**) constraints whose ubiquitous nature calls for classification
- Such “**constraint patterns**” are rectangles, thus easy to draw and recall
- In the next slides we browse a little “constraint bestiary” capturing some typical samples.



# Constraints are Rectangles

- All of shape

$$R \cdot I \subseteq O \cdot R$$

- Example: **referential integrity** in general, where  $N$  is the *offer* and  $M$  is the *demand* :

$$\rho(\epsilon_F \cdot M) \subseteq \delta N \Leftrightarrow$$

$$\Leftrightarrow \epsilon_F \cdot M \subseteq N^o \cdot T$$

$M$ ,  $N$  simple.  $\epsilon_F$  is a membership relation.

# Constraints are Rectangles

- **Example:**  $M$ ,  $N$  domain-disjoint

$$M \cdot N^\circ \subseteq \perp$$

- **Example:** simple  $M$ ,  $N$  domain-coherent

$$M \cdot N^\circ \subseteq id$$

- **Example:**  $M$  domain-closed by  $R$ :

$$M \cdot R^\circ \subseteq T \cdot M$$

(path-closure constraint  $pc$  instance of this)

- **Example:** range of  $R$  in  $\Phi$

$$R \subseteq \Phi \cdot R$$

## Last but not least

- Rectangles enforce **composition** as main relational operator (as in **Alloy**) which is the **multiplicative** operator in the abstract notion of a computation captured by

*Semirings  $(S, +, \cdot, 0, 1)$  inhabited by computations (eg. instructions, statements) where*

- $x \cdot y$  captures **sequencing**
  - $x + y$  captures **choice** (alternation)
  - $0$  means **death**
  - $1$  means **skip** (do nothing)
- Theorem provers such as **Prover9** are especially apt to deal with this kind of structures [3].
- Thus “rectangular” Alloy indeed offers the *other side of the moon* — an effective connection to theorem proving

# What will keep us busy for a while

## Current work:

- Defining a simple pointfree **binary** relational semantics for **Alloy**, hopefully simpler than that of [2] (see appendix)
- (Future: base the conversion of pointwise to PF Alloy on such semantics)
- Studying the translation to/from Haskell and, in particular, how to port counterexamples to QuickCheck.

## Near future:

- Connect **Alloy** to **Prover9** and **Mace4**
- Start the design of an Alloy-centric **tool-chain** incorporating new tools (the MEDEA project)

## Appendix — semantics of “dot join”

Meaning of  $R.S$  in Alloy:

$$\llbracket R.S \rrbracket = \llbracket S \rrbracket \cdot \llbracket R \rrbracket \quad A \xrightarrow{\llbracket R \rrbracket} B \xrightarrow{\llbracket S \rrbracket} C$$

wherever both  $R$ ,  $S$  are binary relations, or

$$\llbracket s.S \rrbracket = \underbrace{\rho(\llbracket S \rrbracket \cdot \llbracket s \rrbracket)}_{sp\ S\ s} \quad B \xrightarrow{\llbracket s \rrbracket} B \xrightarrow{\llbracket S \rrbracket} C \xrightarrow{\llbracket s.S \rrbracket} C$$

wherever  $s$  is a set (unary relation) and  $R$  is binary. (Read  $sp\ R\ s$  as meaning the *strongest post-condition* ensured by  $R$  once pre-conditioned by  $s$ .)

## Appendix — semantics of “dot join”

Since  $s.\sim r$  is equal to  $r.s$  (as postulated in the Alloy book [4]), that is

$$\llbracket S.s \rrbracket = \llbracket s.\sim S \rrbracket$$

holds, then

$$\llbracket S.s \rrbracket = \underbrace{\delta(\llbracket s \rrbracket \cdot \llbracket S \rrbracket)}_{wp\ S\ s}$$

(Read  $wp\ S\ s$  as meaning the *weakest pre-condition* required for  $S$  to ensure  $s$  on its output.)

In case  $R$  is a function  $f$  and  $s$  is a scalar  $x$ , then  $\llbracket x.f \rrbracket$  simply means function application  $f(x)$ .

## Example — dot join associativity

Let us check under what conditions equality

$$(x.r).s = x.(r.s) \quad (9)$$

holds in Alloy.

In case of binary relations we are done.

The case where  $x$  is unary (a set) follows in the next slide, where uppercase letters denote binary relations.

## Example — dot join associativity

$$\llbracket (x.R).S \rrbracket = \rho(\llbracket S \rrbracket \cdot \llbracket (x.R) \rrbracket)$$

$$\Leftrightarrow \quad \{ \text{definition} \}$$

$$\llbracket (x.R).S \rrbracket = \rho(\llbracket S \rrbracket \cdot \rho(\llbracket R \rrbracket \cdot \llbracket x \rrbracket))$$

$$\Leftrightarrow \quad \{ \text{range of composition} \}$$

$$\llbracket (x.R).S \rrbracket = \rho(\llbracket S \rrbracket \cdot (\llbracket R \rrbracket \cdot \llbracket x \rrbracket))$$

$$\Leftrightarrow \quad \{ \text{standard binary relation associativity} \}$$

$$\llbracket (x.R).S \rrbracket = \rho((\llbracket S \rrbracket \cdot \llbracket R \rrbracket) \cdot \llbracket x \rrbracket)$$

$$\Leftrightarrow \quad \{ R \text{ and } S \text{ are binary} \}$$

$$\llbracket (x.R).S \rrbracket = \llbracket x.(R.S) \rrbracket$$



## Example — dot join associativity

Wherever only the middle component is unary, associativity

$$(R.x).S = R.(x.S)$$

holds under side condition  $\sim R.S = S.\sim R$ .

It never holds in case of multiary relations — the equality doesn't even type check!

The general rule, as in Alloy's book [4]:

*If two ways to parenthesize a join expression are both well formed they will be equivalent.*

What does “well formed” mean? Currently formally checking **informal** statements such as this in the book.



P.J. Freyd and A. Ščedrov.

*Categories, Allegories*, volume 39 of *Mathematical Library*.  
North-Holland, 1990.



Marcelo F. Frias, Carlos G. Lopez Pombo, Gabriel A. Baum,  
Nazareno M. Aguirre, and Thomas S.E. Maibaum.

Reasoning about static and dynamic properties in alloy: A  
purely relational approach.

*ACM Trans. Softw. Eng. Methodol.*, 14(4):478–526, 2005.



Peter Höfner and Georg Struth.

On automating the calculus of relations.

In Alessandro Armando, Peter Baumgartner, and Gilles Dowek,  
editors, *IJCAR*, volume 5195 of *Lecture Notes in Computer  
Science*, pages 50–66. Springer, 2008.



D. Jackson.

*Software abstractions: logic, language, and analysis*.

The MIT Press, Cambridge Mass., 2006.

ISBN 0-262-10114-9.



Joost Visser.

Real estate exchange.

Technical report, DI/UM , Braga, Jan 2007.

PortoDigital – SEC-11. Confidential.



P.L. Wadler.

Theorems for free!

In *4th International Symposium on Functional Programming Languages and Computer Architecture*, pages 347–359,  
London, Sep. 1989. ACM.