Quantitative domain theory

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Abstract
One of the nice features of domain theory is the strong interaction between order-theoretic, topological and algebraic ideas. For instance, continuous lattices can be described as ordered sets with certain completeness properties, as injective topological T_0-spaces with respect to embeddings, or as Eilenberg–Moore algebras for the filter monad on Set. Since F.W. Lawvere's famous 1973 paper it is well-known that both ordered sets and metric spaces can be viewed as quantale-enriched categories: the former ones for the quantale $\mathsf{V} = 2$, the latter ones for the quantale $\mathsf{V} = [0, \infty]$. There exist many interesting attempts in the literature to introduce the notion of *continuous metric spaces*, or, more general, *continuous* V*-categories*; usually based on generalisations of the order-theoretic description of continuous lattices. In this talk we will consider an approach to domain theory using "enriched topological spaces". In particular, we obtain a V-enriched equivalent to the filter monad, whose algebras might deserve to be called continuous V-categories.