

# Leis do Cálculo Relacional

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AXIOMS:  $\cdot, \subseteq$

(associativity)	$(R \cdot S) \cdot T = R \cdot (S \cdot T)$
(identity)	$R = R \cdot id = id \cdot R$
( $\subseteq$ reflexivity)	$R \subseteq R$
( $\subseteq$ transitivity)	$R \subseteq S \wedge S \subseteq T \Rightarrow R \subseteq T$
( $\subseteq$ anti-symmetry)	$R \subseteq S \wedge S \subseteq R \Rightarrow R = S$
(monotonicity)	$S \subseteq T \wedge R \subseteq U \Rightarrow S \cdot R \subseteq T \cdot U$

AXIOMS:  $=$

(ping-pong)	$R = S \equiv R \subseteq S \wedge S \subseteq R$
(indirection)	$R = S \equiv \forall X \cdot (X \subseteq R \wedge X \subseteq S)$

AXIOMS:  $\cap, \circ$

( $\cap$ universal)	$X \subseteq (R \cap S) \equiv X \subseteq R \wedge X \subseteq S$
( $\circ$ involution)	$R^{\circ\circ} = R$
( $\circ$ monotonicity)	$R \subseteq S \equiv R^{\circ} \subseteq S^{\circ}$
( $\circ$ contravariance)	$(R \cdot S)^{\circ} = S^{\circ} \cdot R^{\circ}$
(left modular)	$(R \cdot S) \cap T \subseteq R \cdot (S \cap (R^{\circ} \cdot T))$

DEDEKIND VARIATIONS

(left modular)	$(R \cdot S) \cap T \subseteq R \cdot (S \cap (R^{\circ} \cdot T))$
(right modular)	$(R \cdot S) \cap T \subseteq (R \cap (T \cdot S^{\circ})) \cdot S$
(weak distr)	$(R \cdot S) \cap T \subseteq (R \cap (T \cdot S^{\circ})) \cdot (S \cap (R^{\circ} \cdot T))$

$\cap$  LAWS

( $\cap$ associativity)	$(R \cap S) \cap T = R \cap (S \cap T)$
( $\cap$ commutativity)	$R \cap S = S \cap R$
( $\cap$ idempotence)	$R \cap R = R$
( $\cap$ abbreviation)	$R \subseteq S \equiv R = R \cap S$
( $\cap$ cancellation)	$R \cap S \subseteq R, R \cap S \subseteq S$
( $\cap$ left fusion)	$T \cdot (R \cap S) \subseteq T \cdot R \cap T \cdot S$
( $\cap$ right fusion)	$(R \cap S) \cdot T \subseteq R \cdot T \cap S \cdot T$

## ◦ LAWS

(reduction)	$R \subseteq R^\circ \equiv R = R^\circ$
(wrap)	$R \subseteq R \cdot R^\circ \cdot R$
(dist over $\cap$ )	$(S \cap R)^\circ = R^\circ \cap S^\circ$

## MISC

(Dedekind: $T = id, R = R^\circ$ )	$(R^\circ \cdot S) \cap id \subseteq (R \cap S)^\circ \cdot (R \cap S)$
(Dedekind: $S = id$ )	$R \cap T \subseteq R \cdot (id \cap (R^\circ \cdot T))$

## KERNEL LAWS

(definition)	$\ker R = R^\circ \cdot R$
(duality)	$\ker R^\circ = \text{img } R$
(monotonicity)	$R \subseteq S \Rightarrow \ker R \subseteq \ker S$
(symmetry)	$(\ker R)^\circ = \ker R$
(intro)	$R \subseteq R \cdot \ker R$
( $\cdot$ kernel)	$\ker (R \cdot S) = S^\circ \cdot \ker R \cdot S$
( $\cap$ kernel)	$R^\circ \cdot S \cap id \subseteq \ker (R \cap S)$

## IMAGE LAWS

(definition)	$\text{img } R = R \cdot R^\circ$
(duality)	$\text{img } R^\circ = \ker R$
(monotonicity)	$R \subseteq S \Rightarrow \text{img } R \subseteq \text{img } S$
(symmetry)	$(\text{img } R)^\circ = \text{img } R$
(intro)	$R = \text{img } R \cdot R$

## ORDER TAXONOMY

(reflexive)	$id_A \subseteq R$
(coreflexive)	$R \subseteq id_A$
(transitive)	$R \cdot R \subseteq R$
(symmetric)	$R \subseteq R^\circ$
(anti-symmetric)	$R \cap R^\circ \subseteq id_A$
(connected)	$R \cup R^\circ = \top_A$
(entire)	$id \subseteq \ker R$ (total relation)
(simple)	$\text{img } R \subseteq id$ (partial function)
(surjection)	$R^\circ$ entire
(injection)	$R^\circ$ simple

## COREFLEXIVES

(symm & transitive)	$R = R^\circ = R \cdot R = R \cap id$
(cancellation)	$R$ coreflexive $\Rightarrow (R \cdot T \subseteq T) \wedge (T \cdot R \subseteq T)$
(distributivity)	$R$ coreflexives $\Rightarrow (R \cdot T) \cap S = R \cdot (T \cap S)$

## GALOIS CONNECTIONS

Relational Operators as Galois Connections			
$(f \ X) \subseteq Y \equiv X \subseteq (g \ Y)$			
Description	$f = g^\flat$	$g = f^\sharp$	Obs.
converse	$(-)^{\circ}$	$(-)^{\circ}$	
shunting rule	$(f \cdot)$	$(f^{\circ} \cdot)$	NB: $f$ is a function
“converse” shunting rule	$(\cdot f^{\circ})$	$(\cdot f)$	NB: $f$ is a function
left-division	$(R \cdot)$	$(R \setminus \ )$	$R$ under ...
right-division	$(\cdot R)$	$(\ / R)$	... over $R$
range	$rng$	$(\cdot \top)$	lower $\subseteq$ restricted to coreflexives
domain	$dom$	$(\top \cdot)$	lower $\subseteq$ restricted to coreflexives
implication	$(R \cap \ )$	$(R \implies \ )$	Note that $(R \implies \ ) = (\neg R \cup \ )$
difference	$(\_ - R)$	$(R \cup \ )$	
PROPERTIES			
cancellation	$X \subseteq (g \cdot f)X \qquad (f \cdot g)Y \subseteq Y$		
definition	$f \ X = \bigcap \{Y \mid X \subseteq gY\}$	$g \ Y = \bigcup \{X \mid f \ X \subseteq Y\}$	
distribution	$f(X \cup Y) = (f \ X) \cup (f \ Y)$	$g(X \cap Y) = (g \ X) \cap (g \ Y)$	$f(\bigcup_i X_i) = \bigcup_i (f \ X_i)$ $g(\bigcap_i X_i) = \bigcap_i (g \ X_i)$