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# **An Introduction to Formal Modelling**

***DI/UM, 2002-06***

J.N. Oliveira

# Cover Story

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Excerpt of article in the CAMBRIDGE EVENING NEWS:

## Computer Scientist Gets to the "Bottom" of Financial Scandal

*A Cambridge computer professor, Simon Peyton Jones, has made an interesting discovery regarding the **Enron** collapse. (...) Enron's collapse was due to a nearly impenetrable web of financial contracts that disguised the true financial state of the company (...)*

# Cover Story (cont.)

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*(...) Accountants find that even when they are scrupulously honest about the valuation of such contracts there can still be sharp disagreements in regard to the worth of trading reserves, debts, and other components.*

*Enter Professor Peyton Jones. As part of his research at **Microsoft** in Cambridge, he developed a computer language for describing and valuing financial contracts.  
(...)*

# Cover Story (cont.)

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*(...) With colleagues Jean-Marc Eber and Julian Seward, they developed a language capable of **accurately** describing and valuing even the most complex financial instruments. (...)*

*"While accountants find financial derivatives to be mysterious and difficult, for us they are just ordinary **recursive equations**,"*

*says Jones.*

# Cover Story (cont.)

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*(...) "We have been dealing with these for many years and have developed a wide range of techniques for handling them."*

*(...) According to Peyton Jones, his success in the financial world comes from years of research in **Haskell** (...)*

*"Without the tools developed by the **Haskell** community I would never have been able to do what I've done. It's a jolly wonderful way to program computers"*

*he stated. (...)*

# Cover Story (conclusion)

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(...)

*The Arthur Anderson accounting firm is rumored to have made overtures to Peyton Jones. (...) But Professor Peyton Jones plans to remain where he is.*

*"I'm flattered that my research has finally been of use to someone but I'm quite happy working on **Haskell**. Besides, I don't want to have to wear a suit to work every day."*

# Cover Story (conclusion)

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*...our Anderson accounting firm .  
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(CAMBRIDGE EVENING NEWS,

# Cover Story (conclusion)

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(...)

*Arthur Anderson accounting firm rumored to have made overtures to Jones. (...) But Professor Peyton Jones plans to remain where he is.*

*"I'm flattered that my research has really been of use to some of the people."*

(CAMBRIDGE EVENING NEWS, 1st of April (!) 2002)



# Prof. Peyton Jones' “magic words”

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- ... language capable of **accurately** describing and valuing ...
- ... just ordinary recursive **equations** ...
- ... tools developed by the **Haskell** community

In other words:

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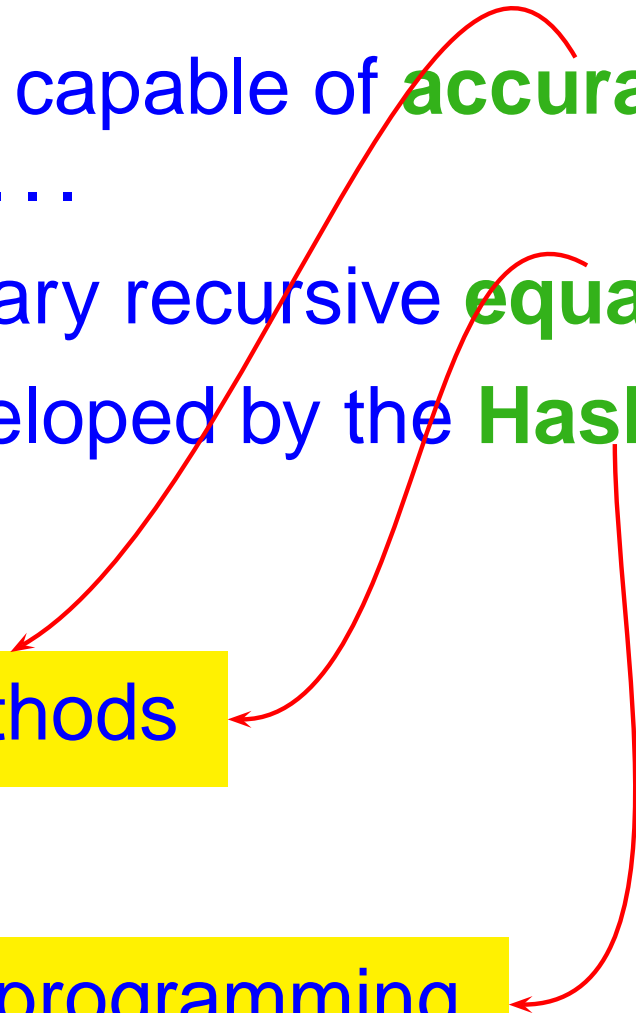
- formal methods

# Prof. Peyton Jones' “magic words”

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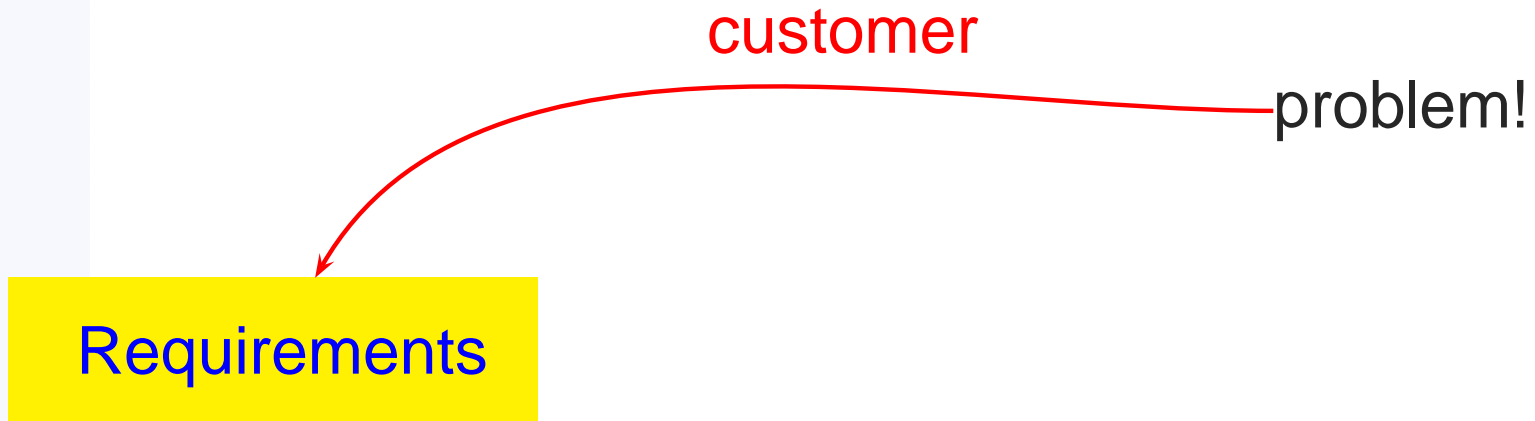
- ... language capable of **accurately** describing and valuing ...
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In other words:

- **formal methods**
  - and
  - **functional programming**
- 

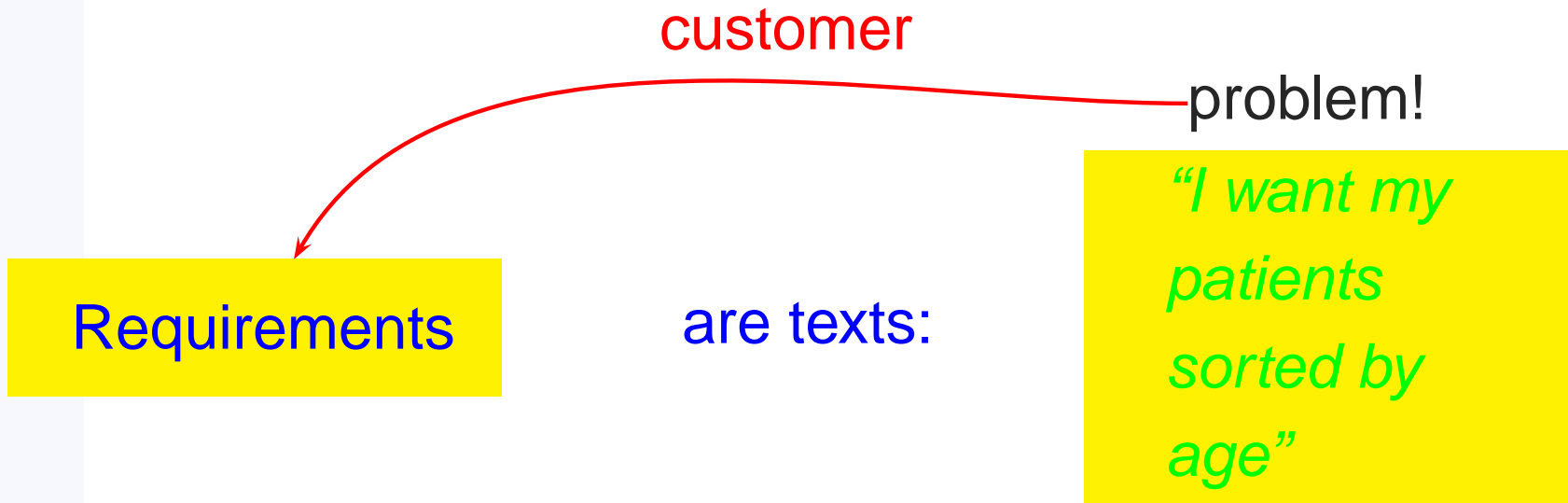
# Notion of specification

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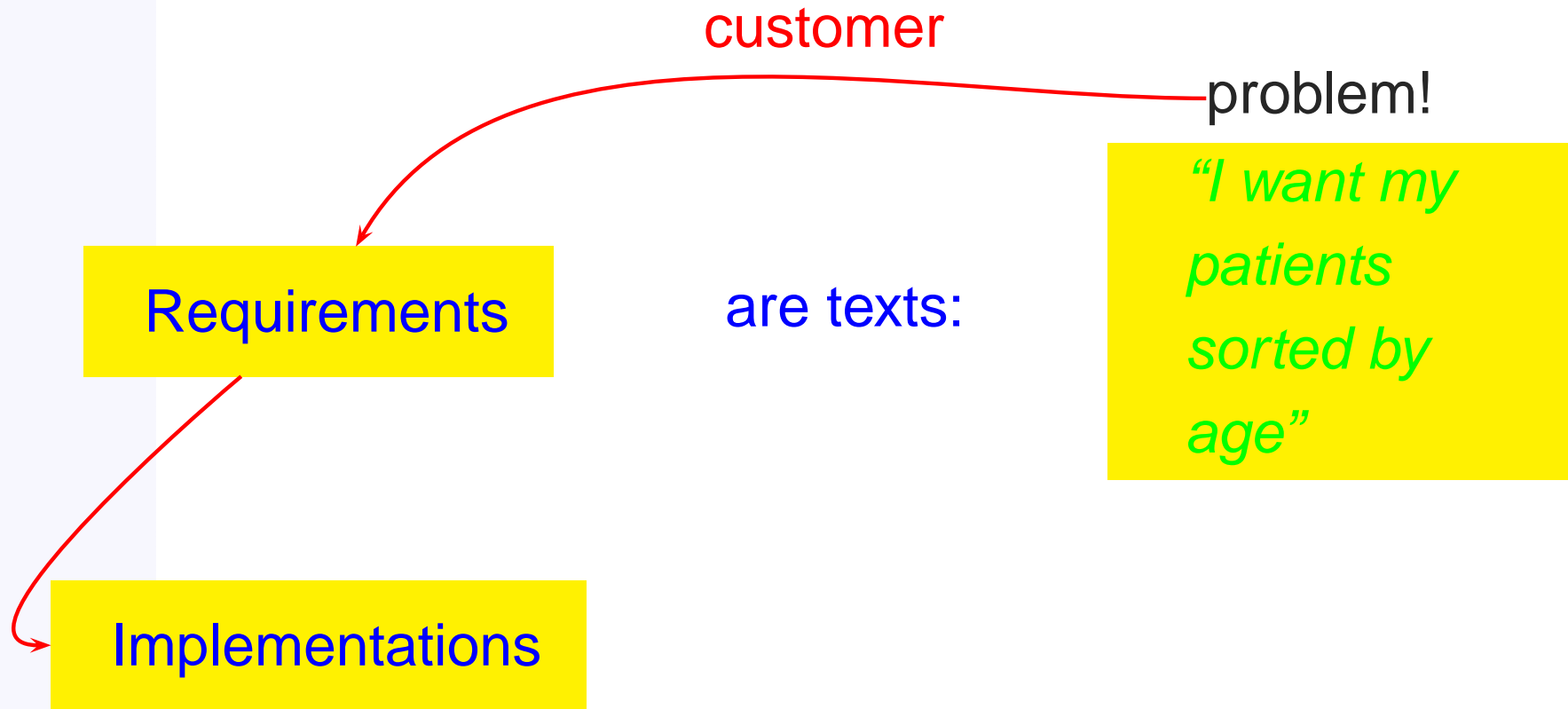
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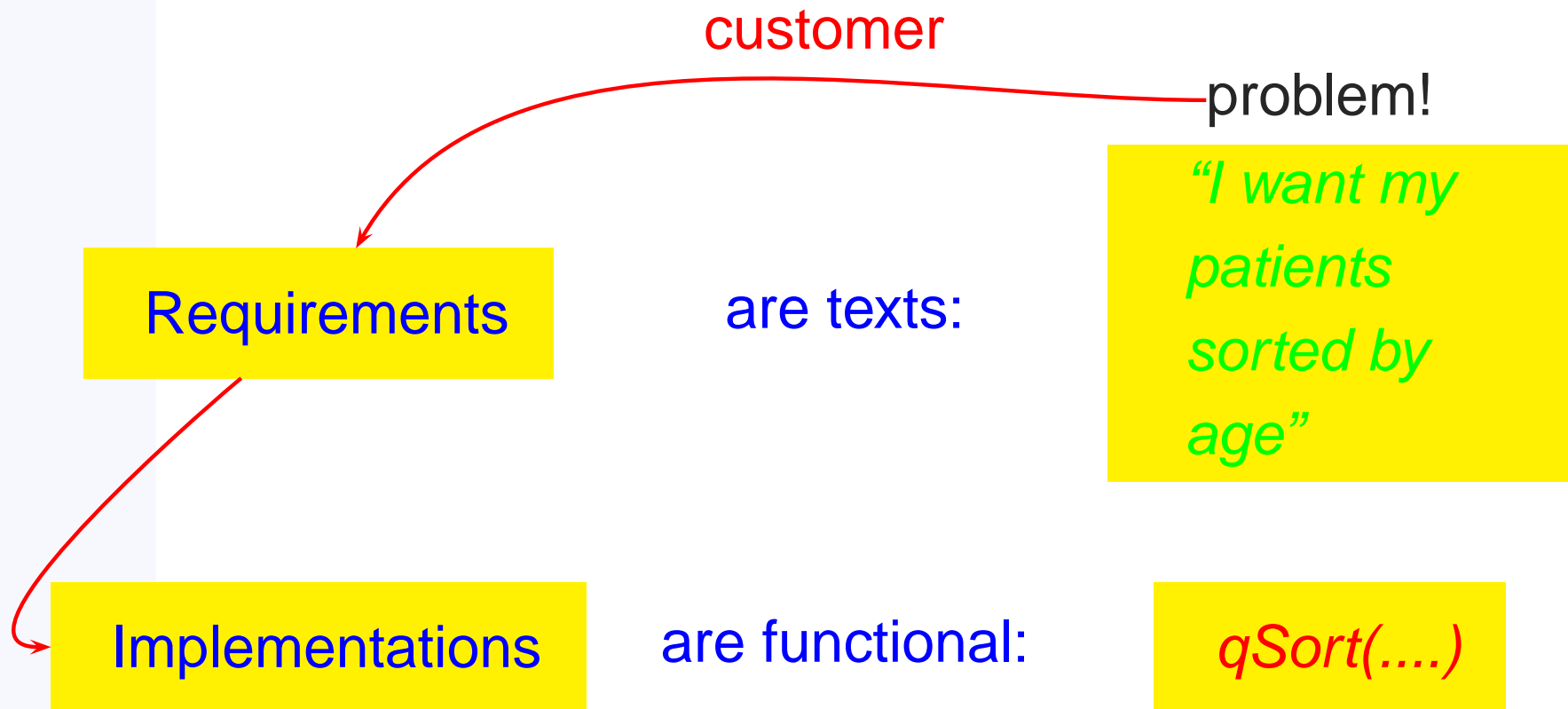
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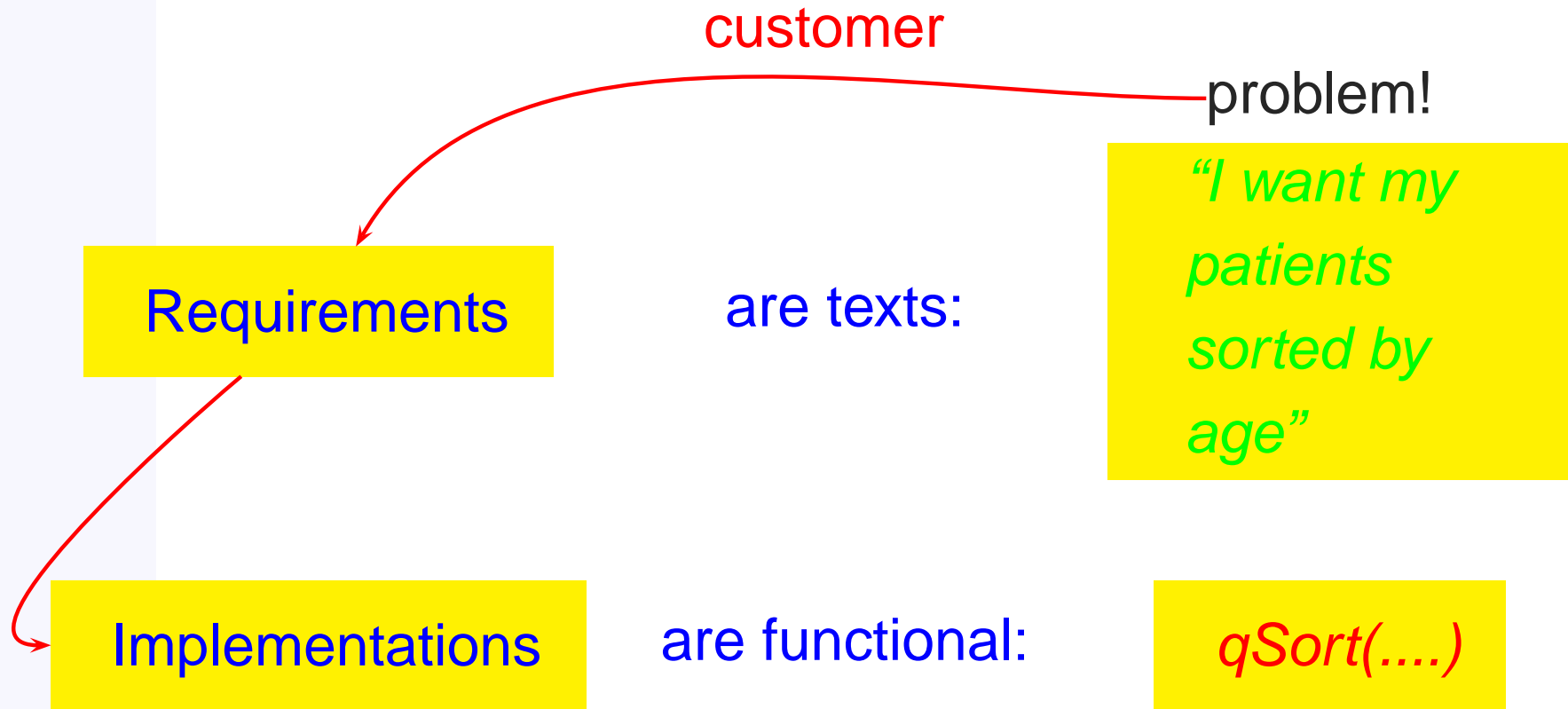
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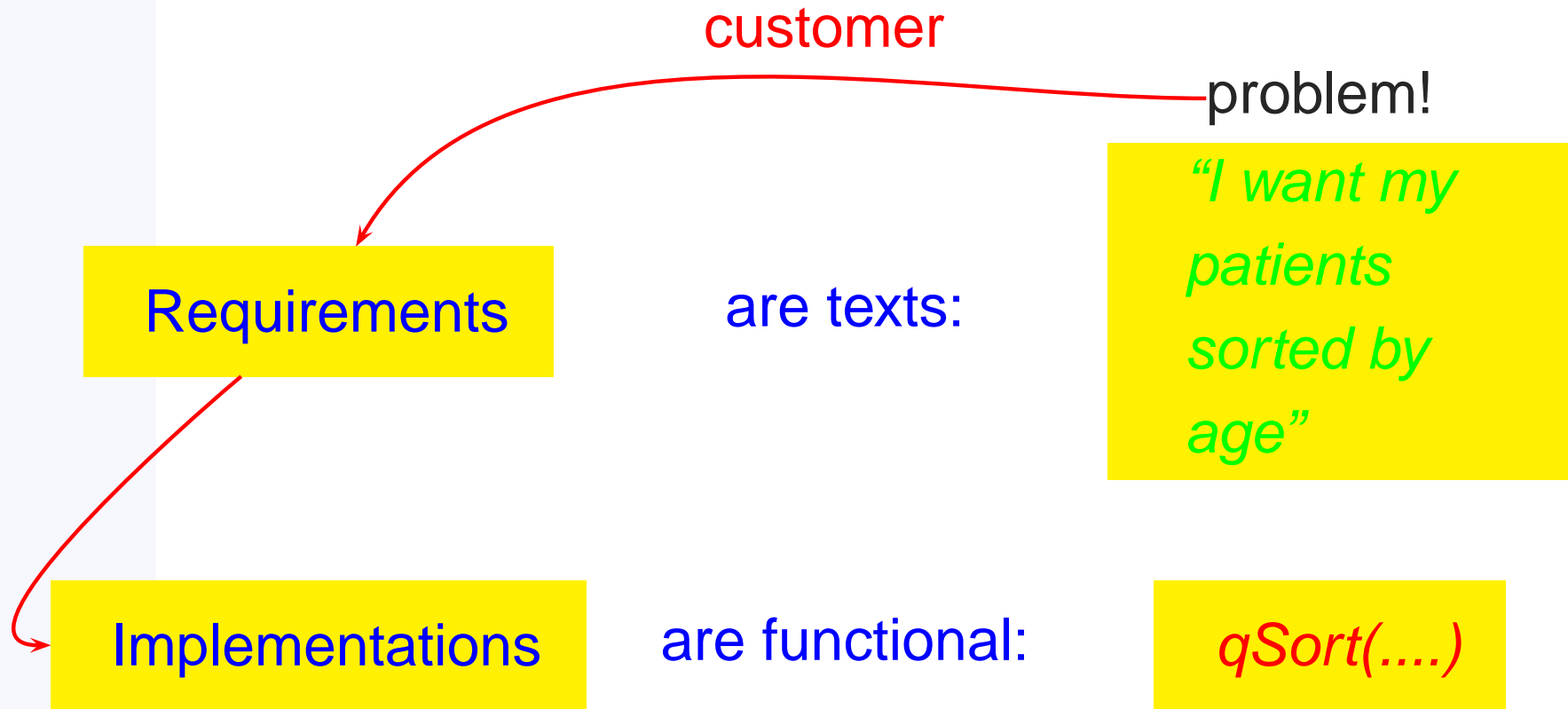


How do we accurately describe **sorting** without mentioning any sorting **algorithm**?



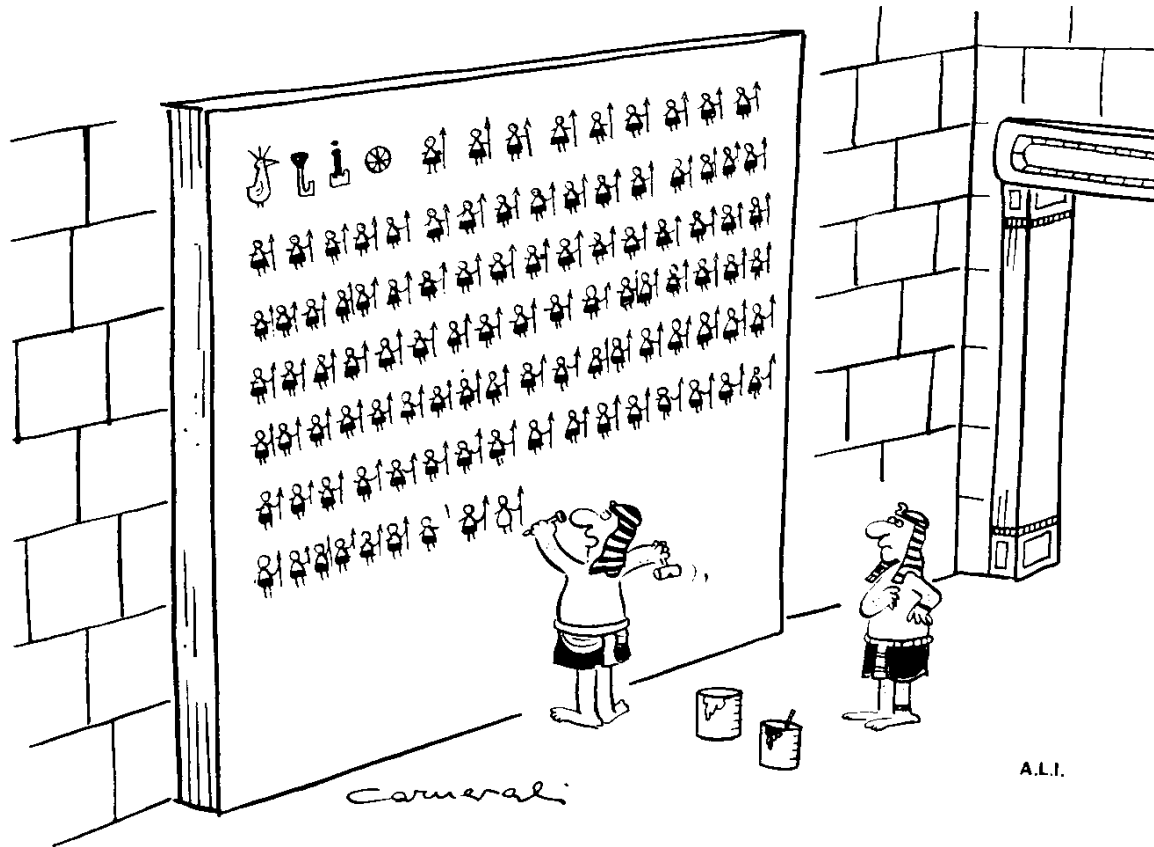
# Notion of specification

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How do we accurately describe sorting without mentioning any sorting algorithm? We need a specification language.

# Why accurate (formal) notations?



*Are you sure there isn't a simpler means of writing  
'The Pharaoh had 10,000 soldiers?'*

# Trend for Notation Economy

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- **Notation** — always a concern throughout the history of mathematics.
- In the 16th century,

$$12x^3 + 18x^2 + 27x + 17$$

would be written

12.cu.~.18.ce.~.27.co.~.17

— cf. **Libro de algebra** (1567) by Coimbra mathematician **Pedro Nunes** (1502-1578).

- Such notation was at its time replacing a even more obscure syntax.

# Requirement analysis

---

From a mobile phone manufacturer:

(...) For each list of calls stored in the mobile phone (eg. numbers dialed, SMS messages, lost calls), the store operation should work in a way such that (a) the more recently a call is made the more accessible it is; (b) no number appears twice in a list; (c) only the last 10 entries in each list are stored.

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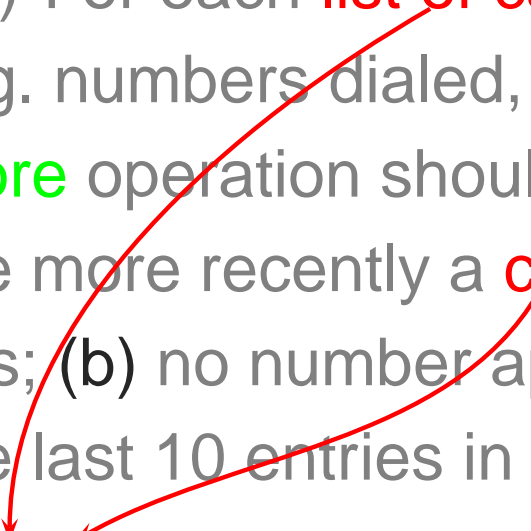
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**data-type** (= “noun”);

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**data-type** (= "noun");

**function** (= "verb");

**property** (= "integrated sentence");



# Formal model

---

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In **Haskell** notation:

```
store :: Call -> [Call] -> [Call]
store c l = ...
```

# Formal model

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In **VDM-SL** notation:

```
store : Call -> seq of Call -> seq of Call  
store (c)(1) = ...
```

# Meeting the requirements

---

(...) such that (a) the more recently a **call** is made the more accessible it is; (b) no number appears twice in a list; (c) only the last 10 entries in each list are stored.

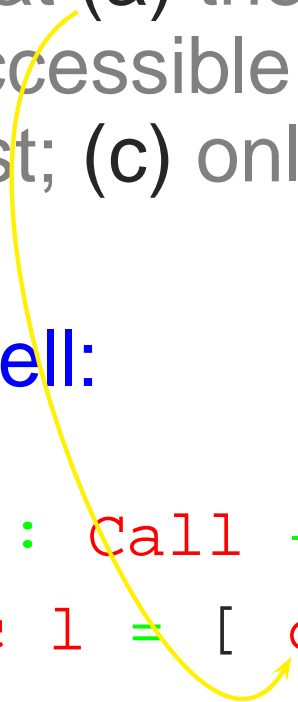
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In Haskell:

```
store :: Call -> [Call] -> [Call]
store c l = [ c ] ++ l
```



---

**Notation:**  $x ++ y$  means “ $x$  catenated with  $y$ ”, eg.


$[ c ] ++ [ a, b, c ] = [ c, a, b, c ]$

# Meeting the requirements

---

(...) such that (a) the more recently a **call** is made the more accessible it is; (b) no number appears twice in a list; (c) only the last 10 entries in each list are stored.

```
store :: Call -> [Call] -> [Call]
store c l = [ c ] ++ filter (/=c) l
```



---

**Notation:** From the **Haskell** Prelude:

```
filter :: (a -> Bool) -> [a] -> [a]
filter p l = [ a | a <- l, p a ]
```

# Meeting the requirements

---

(...) such that (a) the more recently a **call** is made the more accessible it is; (b) no number appears twice in a list; (c) only the last 10 entries in each list are stored.



```
store'  :: Call -> [Call] -> [Call]
store'  c l = take 10 (store c l)
```

---

## Notation:

```
take 0 _ = [] take _ [] = []
```

```
take n (x:xs) | n>0 = x : take (n-1) xs
               | otherwise = []
```



# Writing it in VDM-SL

---

```
store' : Call -> seq of Call -> seq of Call
store'(c)(l) = take(10)(store(c)(l));
store : Call -> seq of Call -> seq of Call
store (c)(l) = [ c ] ^
                [ a | a <- l & a <> c ];
```

---

**Notation:**  $x^y$  is the VDM-SL equivalent of  $x \mathrel{++} y$ .

Notation  $[ a \mid a <- l \ \& \ a <> c ]$  is not valid **VDM-SL**. One has to write

$$[ l(i) \mid i \text{ in set inds } l \ \& \ l(i) <> c ].$$

# Common practice, in eg. C<sup>#</sup>

---

```
public void store10(string phoneNumber)
{
    System.Collections.ArrayList auxList =
        new System.Collections.ArrayList();
    auxList.Add(phoneNumber);
    auxList.AddRange(
        this.filteratmost9(phoneNumber) );
    this.callList = auxList;
}
```

# C# version of store (cont.)

---

```
public System.Collections.ArrayList filteratmost9(string n)
{
    System.Collections.ArrayList retList =
        new System.Collections.ArrayList();
    int i=0, m=0;
    while((i < this.callList.Count) && (m < 9))
    {
        if ((string)this.callList[i] != n)
        {
            retList.Add(this.callList[i]);
            m++;
        }
        i++;
    }
    return retList;
}
```

# Comments on C<sup>#</sup> code

---

Even tolerating code verbosity ...

- How “good” is this implementation?

# Comments on C# code

---

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- How “good” is this implementation?
- Does it meet the 3 properties stated by the mobile phone manufacturer?

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Obs.:

- The same requirements in an FM exam paper led to 5 kinds of answer, of which only one (!) was correct!

# Comments on C# code

---

Even tolerating code verbosity ...

- How “good” is this implementation?
- Does it meet the 3 properties stated by the mobile phone manufacturer?

Obs.:

- The same requirements in an FM exam paper led to 5 kinds of answer, of which only one (!) was correct!
- Alternatively, FMs provide for correct program construction, eg. by calculation.

# Programming by calculation

---

```
store'(c)(l)
= take(10)(store(c)(l))
= take(10)([c]^[l(i)|i in set inds l & l(i)<>c])
= [c]^take(9)([c]^[l(i)|i in set inds l&l(i)<>c])
= [c]^filteratmost(9)(...l...)
= ...
```

---

**Notation:** calculation stems from formal properties, eg.

$$\text{take}(m)(x^y) = \text{take}(m)(x) \wedge \text{take}(m-\text{len } x)(y)$$



# FMs = true software engineering

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How

(implementation)

# FMs = true software engineering

---

What

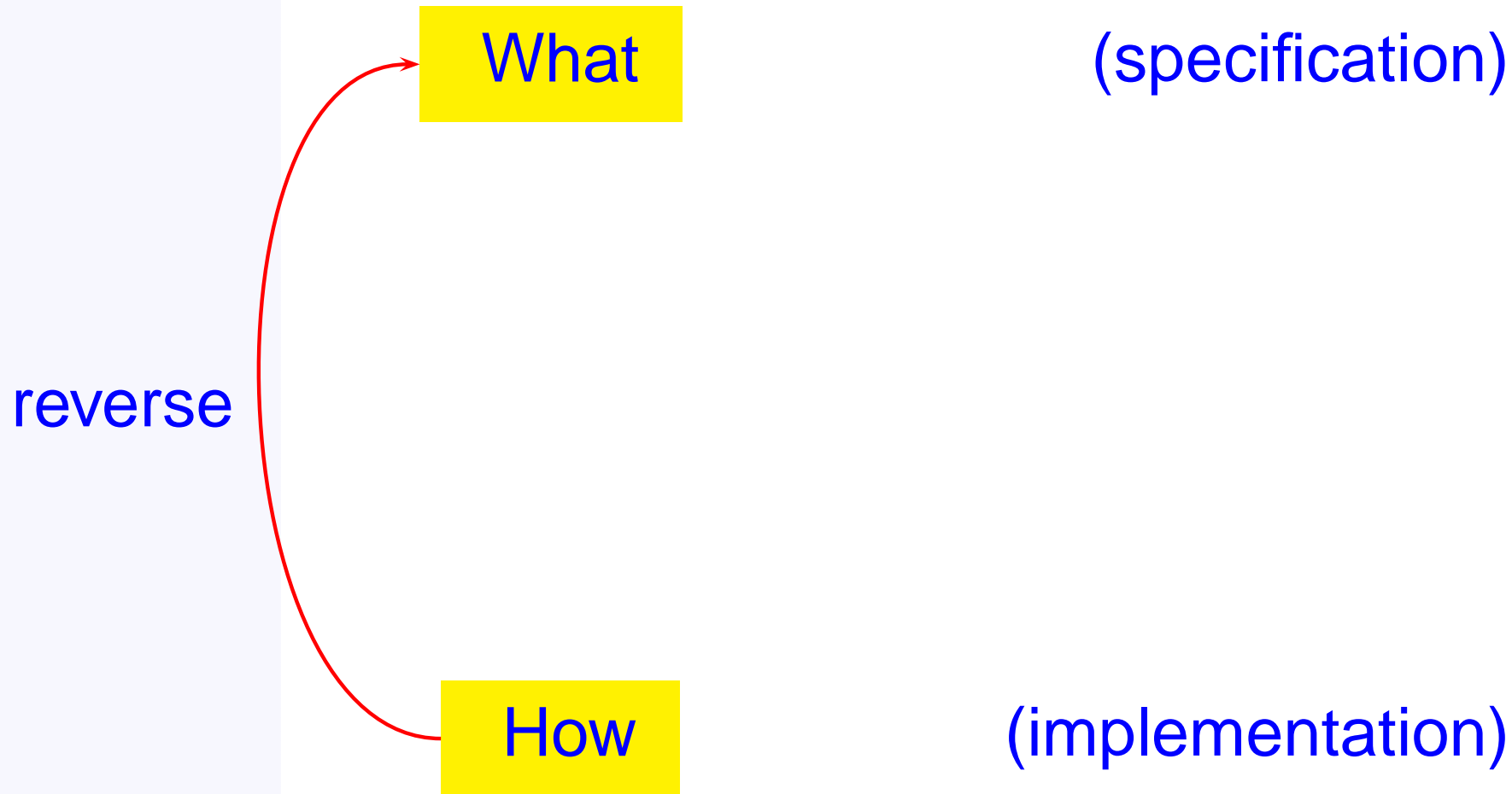
(specification)

How

(implementation)

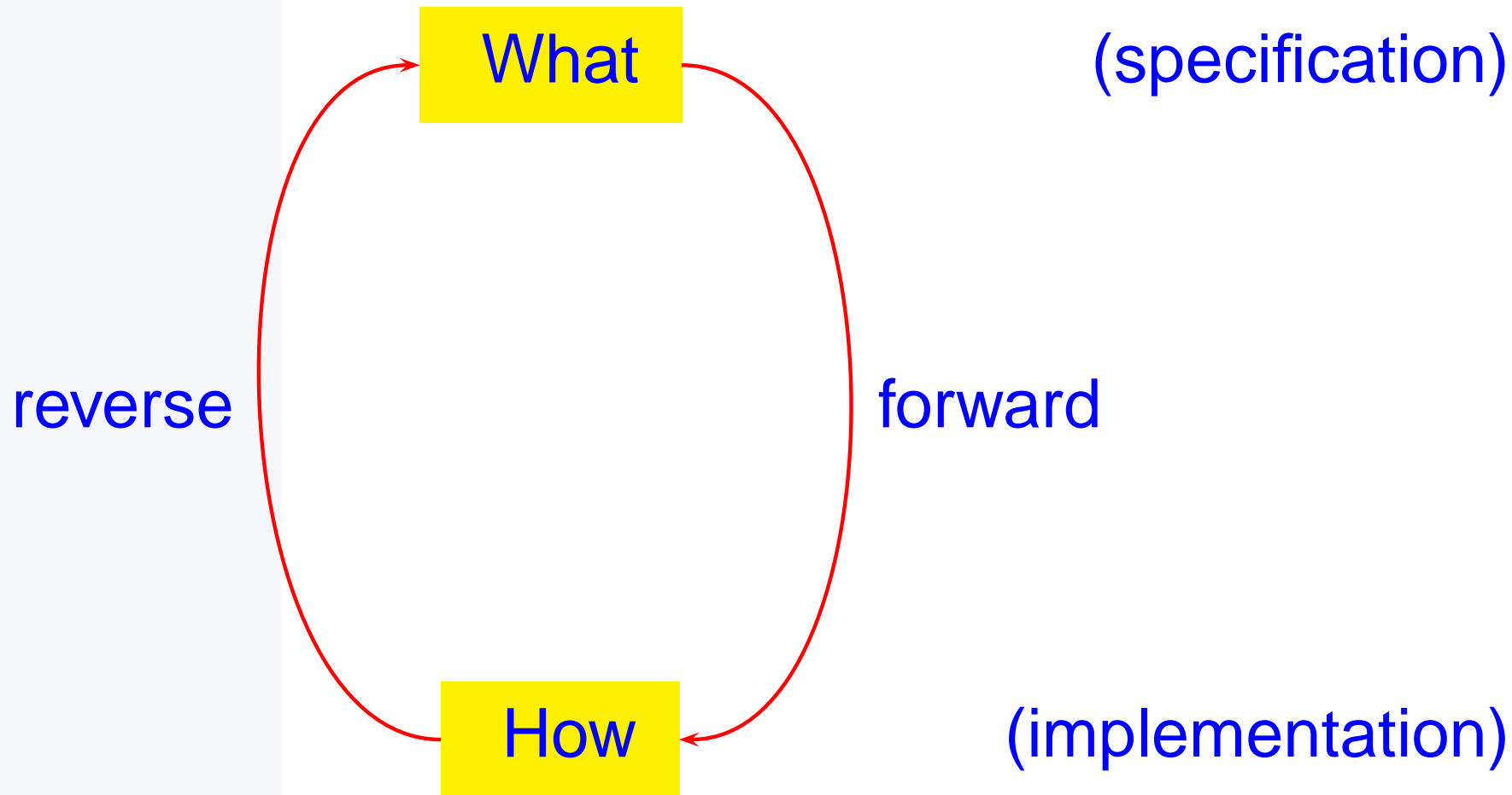
# FMs = true software engineering

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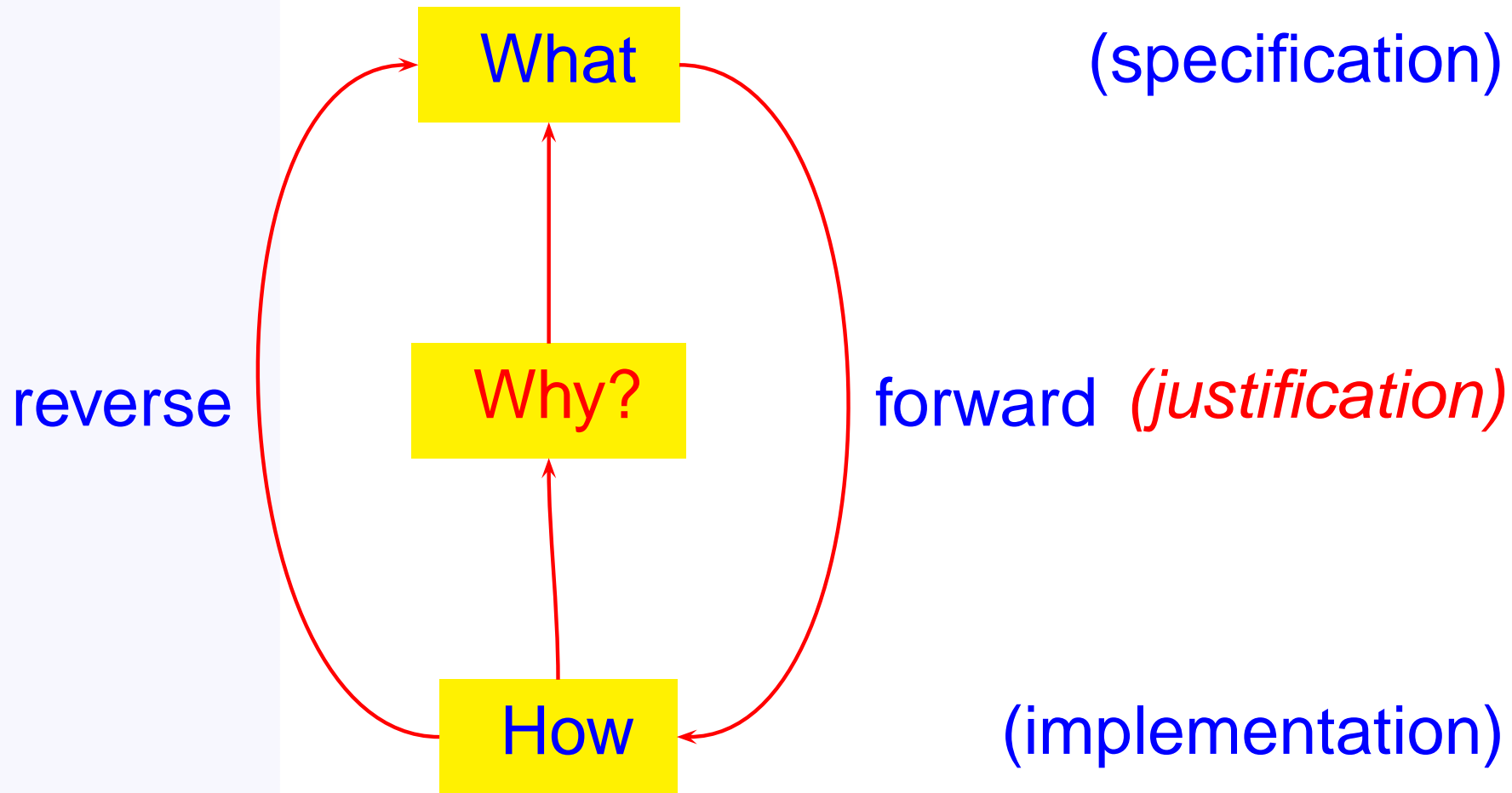
# FMs = true software engineering

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# Balzer's FM Life-cycle

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customer

problem!

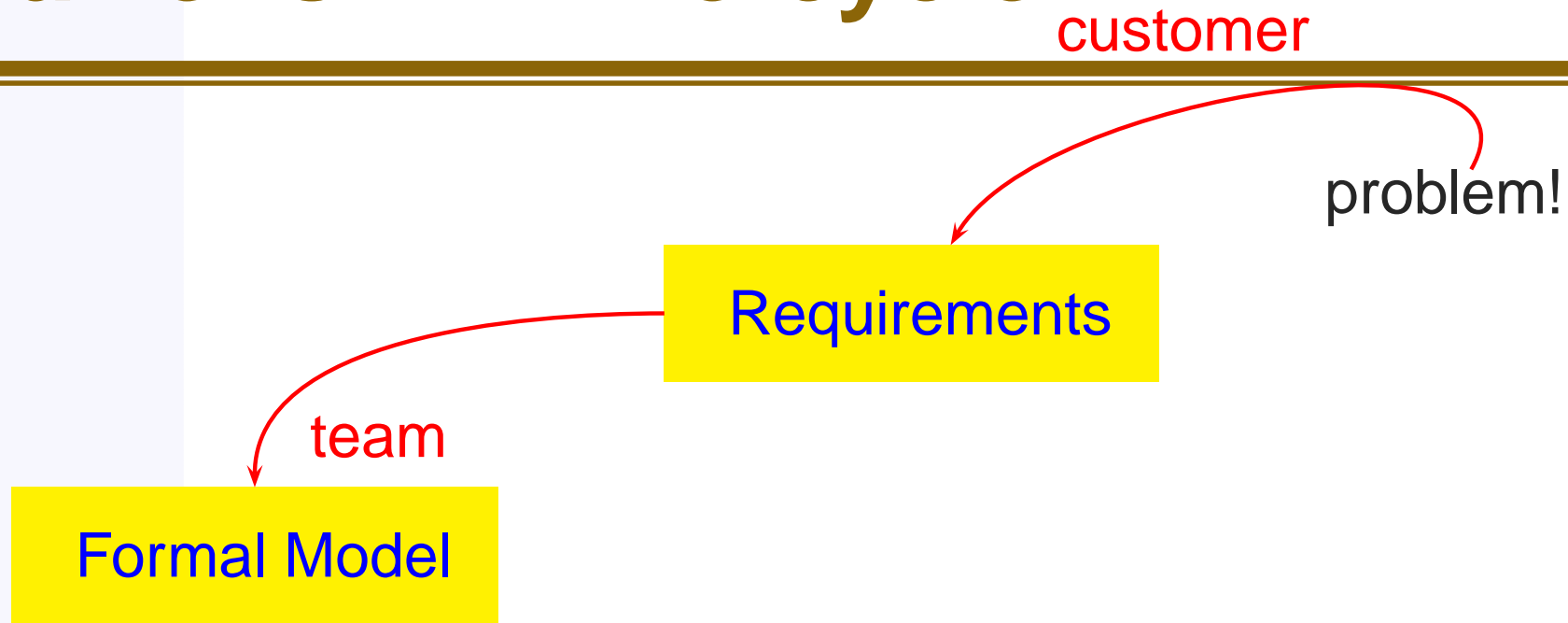


Requirements

The diagram illustrates the initial phase of Balzer's FM Life-cycle. A red curved arrow originates from the word 'problem!' and points to a yellow rectangular box containing the word 'Requirements'. The word 'customer' is positioned above the arrow's starting point.

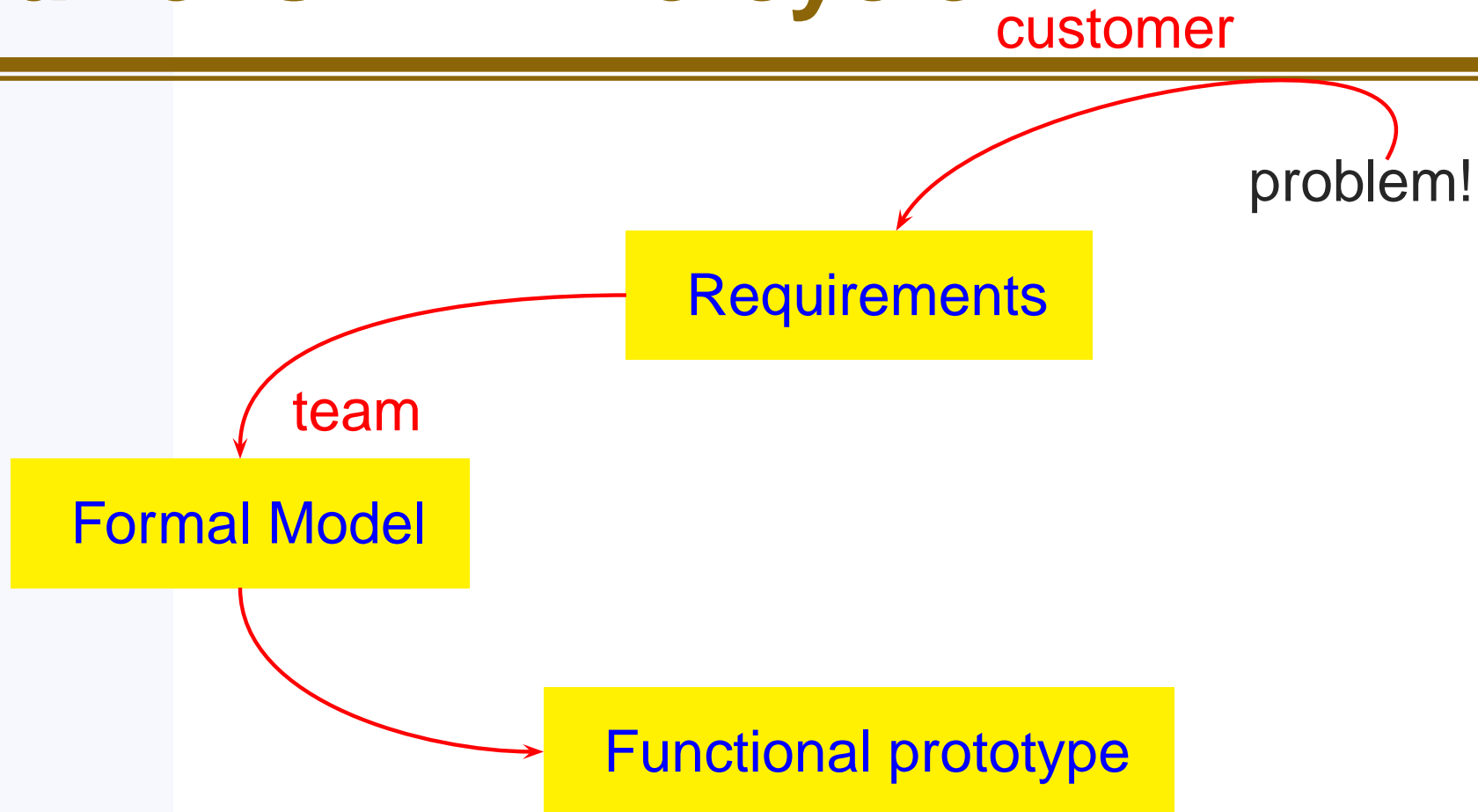
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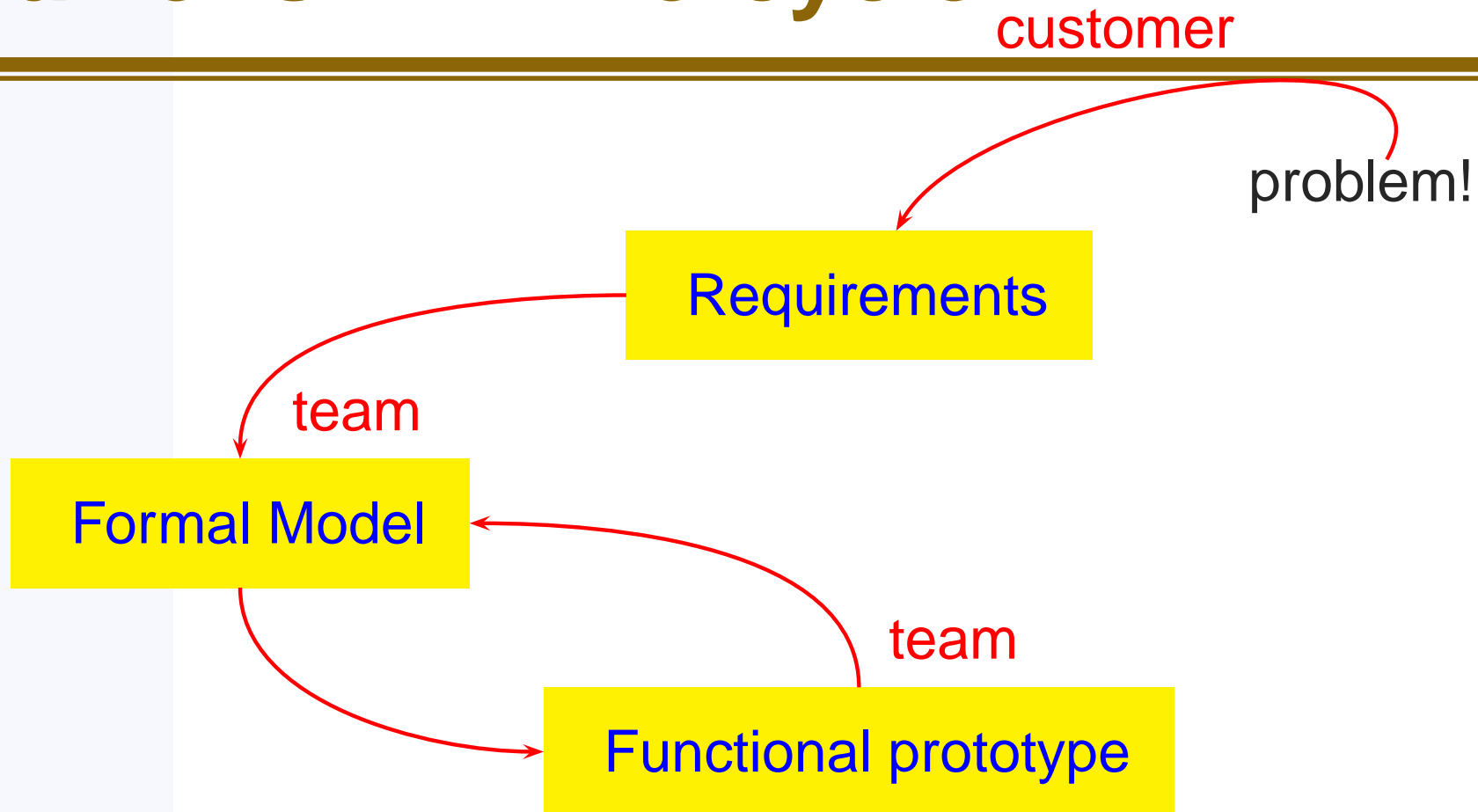
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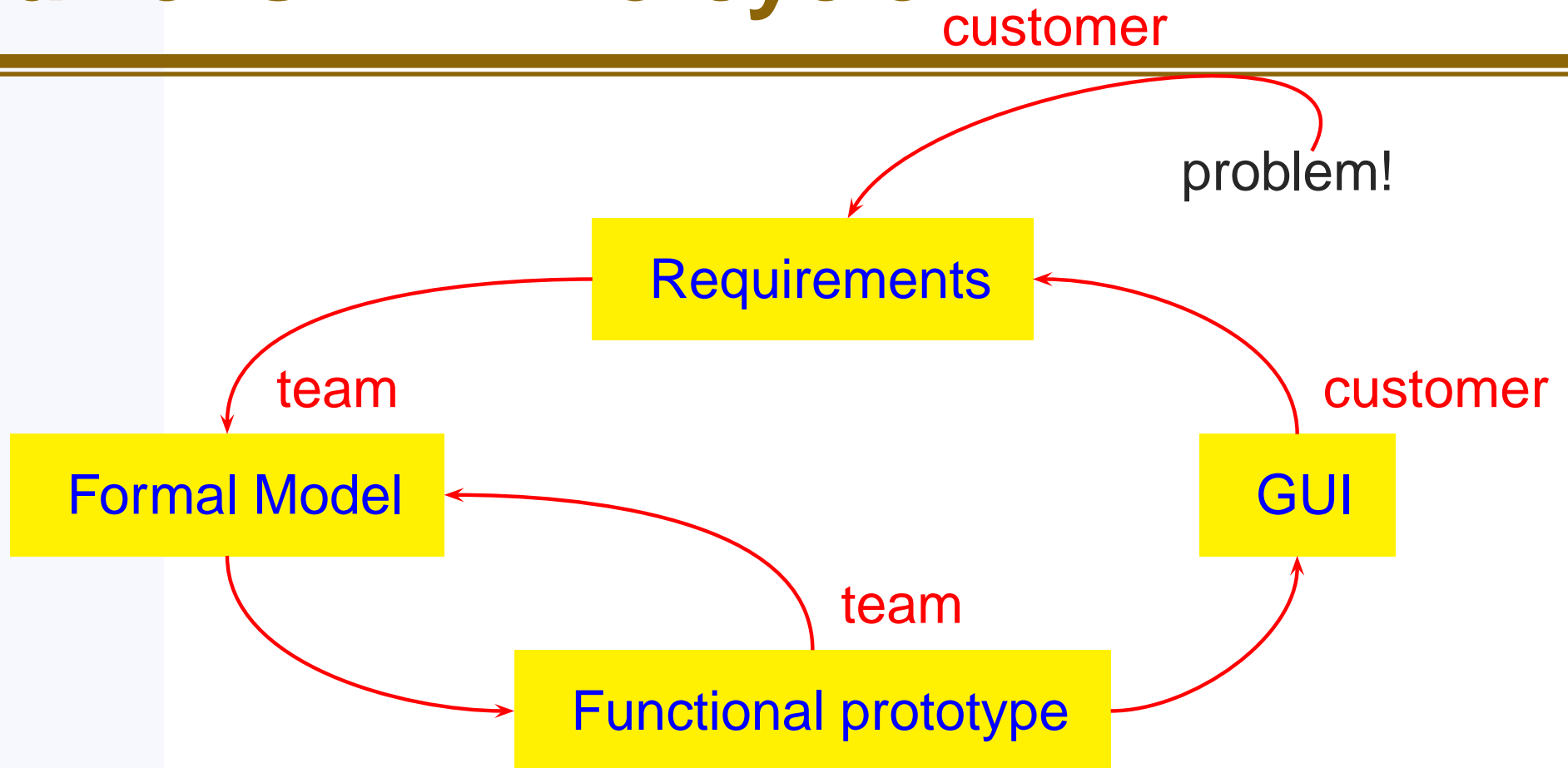




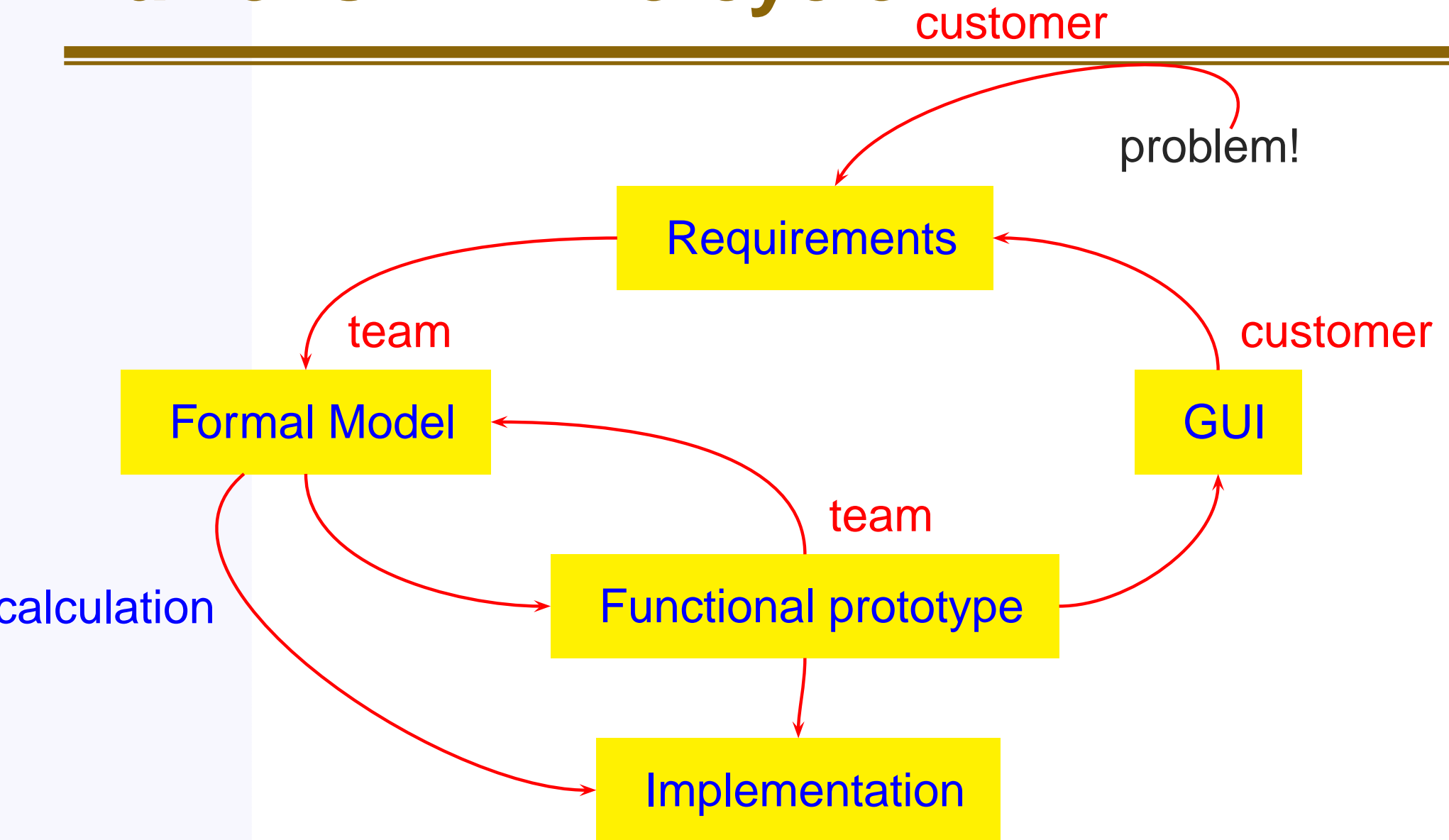
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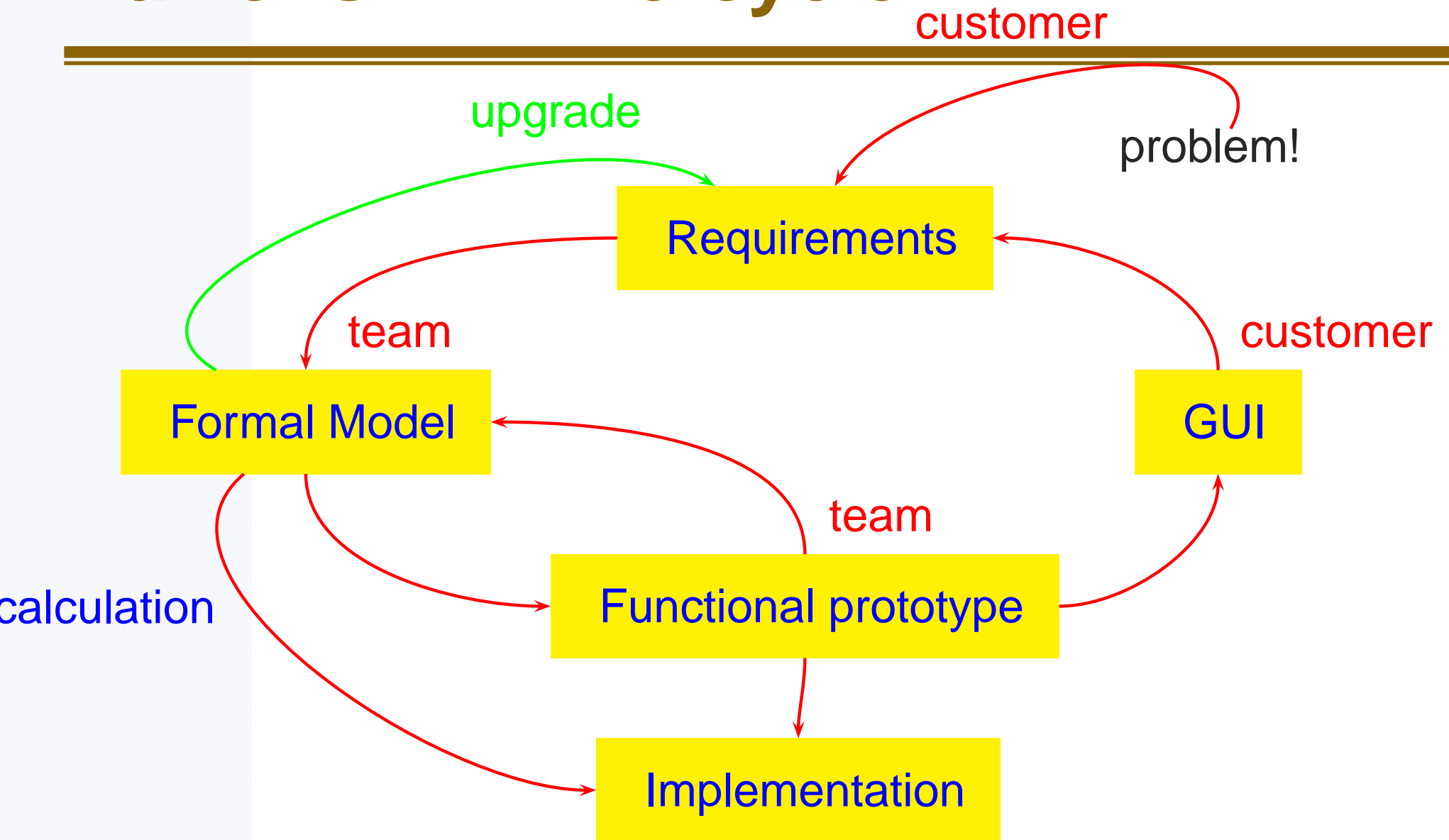
# Balzer's FM Life-cycle



# Balzer's FM Life-cycle



# Balzer's FM Life-cycle



# Our background

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- By 2004:  
20 years of **FM teaching** at the Univ. of Minho
- $\simeq$  10 years ago:  
**Industrial** application of FMs based on FP tested at INESC-BRAGA
- Spin-off of INESC-BRAGA (1996):  
**SIDEREUS S.A.** - Rigorous Solutions for Software Systems (Porto)

# FMs add to competitiveness

---

Increased productivity:

Code Validation {  
    Debug  
    Verification  
    **Calculation** / automation

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Technology-independent documentation:

- the actual enterprise's **wealth**
- **investment** safeguard.

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trains **competitive** software designers :-)