Decomposition of Database Preferences on the Power Set of the Domain

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Motivation

- Database preferences construct strict partial orders
- They are a slight generalization of *Skylines queries*
- The allow optimizing w.r.t. many dimensions simultaneously
- For example: Pareto optimal cars with low fuel consumption (high mpg value) and high power

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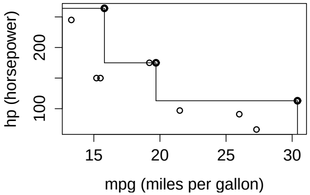

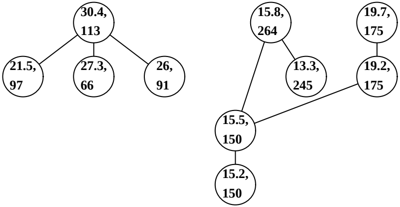

Power set preferences

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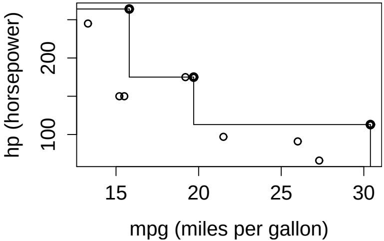
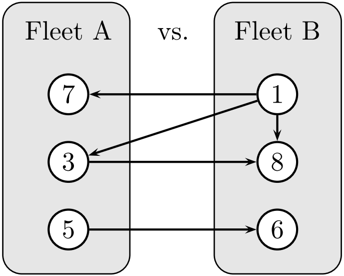




- Hence we search for the *induced power set preference*

Notation and formal background

Notational conventions:

- $r = x_1 + \ldots + x_n$ is a **finite** data set with tuples x_i.
- a preference a has an associated equivalence relation s_a
- relational operations: union $+$, composition $\cdot$, intersection $\sqcap$
- inclusion order $\leq$
- special relations:
  - empty relation 0
  - identity $\mathbb{1}$
  - universal relation $\top$

Preference prerequisites

For preferences (strict orders) a, b we define

- $a \otimes b$ is the *Pareto composition*
  (*"strictly better in (at least) one dimension, better or equal in all dimensions"*)

$$a \otimes b =_{df} (a + \mathsf{s}_a) \sqcap b + a \sqcap (b + \mathsf{s}_b)$$

- The *Prioritisation* $a \,\&\, b$ equals the lexicographical order

$$a \,\&\, b =_{df} a + \mathsf{s}_a \sqcap b$$

- The *set preference* $\mathsf{t}(s)$ for $s \leq r$

$$\mathsf{t}(s) =_{df} \neg s \cdot \top \cdot s$$

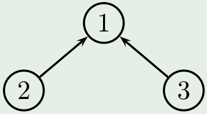
For $|s| = 1$ this is the special case of a *tuple preference*.

Some simple examples of preferences

Example (Some terms over $\mathtt{t}(\cdot)$, $\otimes$, &)

Let $r = x_1 + x_2 + x_3$ a data set. (i) is short for x_i.

- $\mathtt{t}(x_1)$
- $\mathtt{t}(x_1)\ \&\ \mathtt{t}(x_2 + x_3)$



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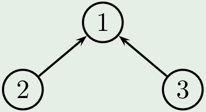

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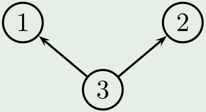

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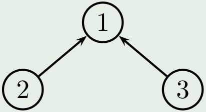

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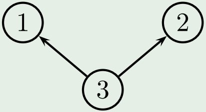

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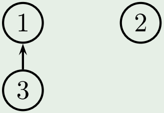

Preferences decomposition

Some results from our MPC'15 paper *"Preference Decomposition and the Expressiveness of Preference Query Languages"*:

- Set preferences and $\otimes$ suffice to express arbitrary strict orders
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In this paper:

- We formally introduce the *power construction* for preferences
- We apply the decomposition algorithms to them
- We introduce some optimizations to retrieve shorter terms

Different power set preferences

Definition (Power set preference)

Let a be a preference, r a data set and $\mathcal{P}(r) = \{p \mid p \leq r\}$ the power set. For all $u, v \in \mathcal{P}(r)$ we define:

$$u \, \pi_0^a \, v \quad \Leftrightarrow_{df} \quad v \neq 0 \quad \wedge \quad \forall y \in v : \exists x \in u : x \, a \, y$$

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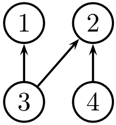

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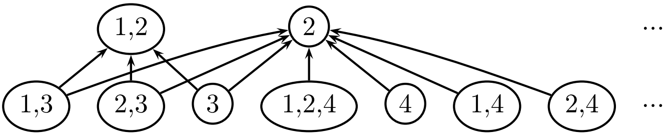
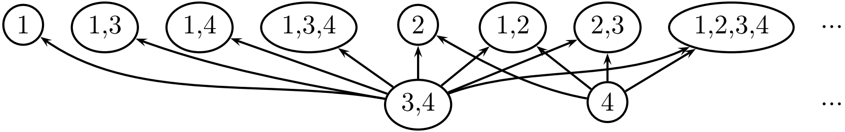


2) Partial graph of π_0^a



3) Partial graph of π_1^a

Simplifications and a third power set preference

The definitions of π_0 and π_1 can be simplified using

$$\langle a|p =_{df} \{(t,t) \in \mathbb{1} \mid \exists t' \in D : (t',t) \in (p \cdot a)\} \qquad \text{(image)},$$
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We get

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for all $u, v \in \widehat{r}$ where

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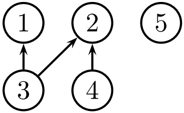
Additionally we define

$$\pi_2^a =_{df} \pi_0^a \sqcap \pi_1^a .$$

The π_i preferences are the three common ways for the power construction of a strict order (cf. Brink, Rewitzky 2001 *"A paradigm for Program Semantics – Power Structures and Duality"*).

- In the following we revisit the decomposition methods from the MPC'15 paper
- Let $r = x_1 + \ldots + x_5$ a data set and consider the following preference:



where ⓘ is short for x_i.

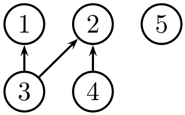
- We show example runs of both preference decomposition methods
- Additional definition: The maximum operator is given by

$$a \rhd r =_{df} r - |a\rangle r \ .$$

Pareto decompositions of set preferences (example run)

$$\text{DECOMP\_PARETO}(a, r) =_{df} \bigotimes_{x \in r} \mathsf{t}(r \cdot \langle a + \mathbb{1} | x \rangle), \quad r = x_1 + \ldots + x_5$$

given preference a



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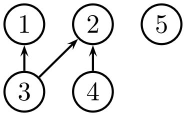

constructed preference b

$b = 0$

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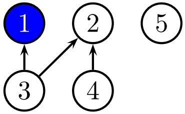

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$$b = 0 \otimes \mathsf{t}(|a + \mathbb{1}\rangle x_1)$$

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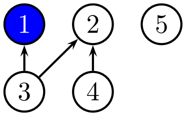

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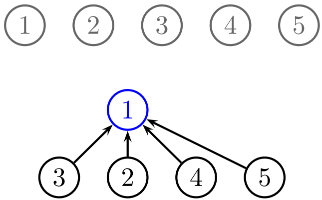

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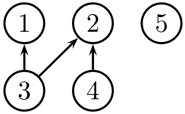

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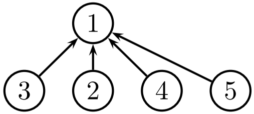

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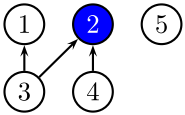

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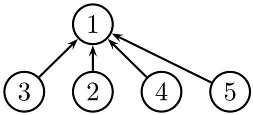

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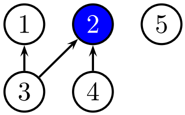

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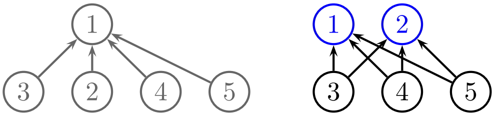

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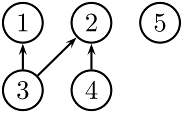

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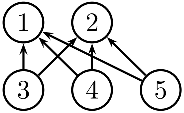

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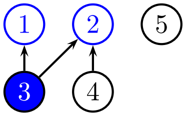

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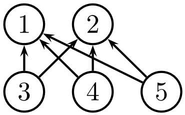

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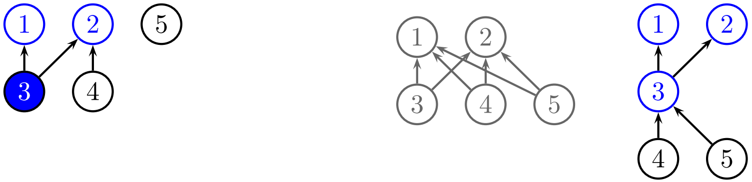

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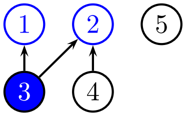

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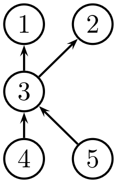

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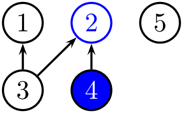

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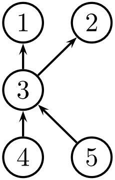

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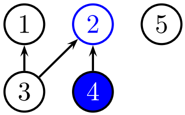

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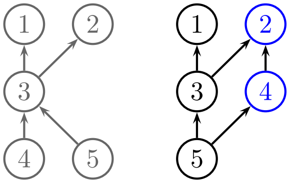

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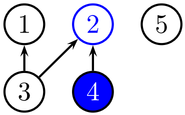

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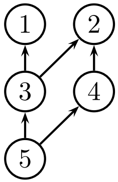
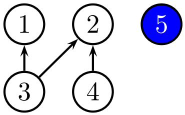


$$b = \mathsf{t}(x_1) \otimes \mathsf{t}(x_2) \otimes \mathsf{t}(x_1 + x_2 + x_3) \otimes \mathsf{t}(x_3 + x_4)$$

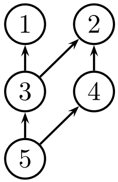
Pareto decompositions of set preferences (example run)

$$\textsc{Decomp\_Pareto}(a, r) =_{df} \bigotimes_{x \in r} \mathsf{t}(r \cdot \langle a + \mathbb{1} | x), \quad r = x_1 + \ldots + x_5$$

given preference a



constructed preference b

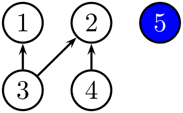


$$b = \mathsf{t}(x_1) \otimes \mathsf{t}(x_2) \otimes \mathsf{t}(x_1 + x_2 + x_3) \otimes \mathsf{t}(x_3 + x_4) \otimes \mathsf{t}(|a + \mathbb{1}\rangle x_5)$$

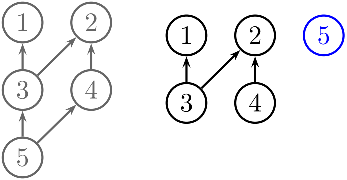
Pareto decompositions of set preferences (example run)

$$\text{DECOMP\_PARETO}(a, r) =_{df} \bigotimes_{x \in r} \mathsf{t}(r \cdot \langle a + \mathbb{1} | x), \quad r = x_1 + \ldots + x_5$$

given preference a



constructed preference b

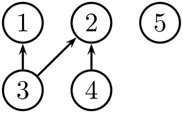


$$b = \mathsf{t}(x_1) \otimes \mathsf{t}(x_2) \otimes \mathsf{t}(x_1 + x_2 + x_3) \otimes \mathsf{t}(x_3 + x_4) \otimes \mathsf{t}(x_5)$$

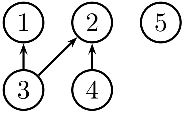
Pareto decompositions of set preferences (example run)

$$\textsc{Decomp\_Pareto}(a, r) =_{df} \bigotimes_{x \in r} \mathsf{t}(r \cdot \langle a + \mathbb{1} | x \rangle), \quad r = x_1 + \ldots + x_5$$

given preference a



constructed preference b



is r-equivalent to a

$$b = \mathsf{t}(x_1) \otimes \mathsf{t}(x_2) \otimes \mathsf{t}(x_1 + x_2 + x_3) \otimes \mathsf{t}(x_3 + x_4) \otimes \mathsf{t}(x_5)$$

Decompositions into tuple preferences (example run)

function DECOMP_TUPLE(a, r)

$a_h \leftarrow a \sqcap \overline{a^2}$

$b[r] \leftarrow 0$

$m \leftarrow a \triangleright r$

while $m \neq 0$ **do**

for all $y \in m$ **do**

$b[y] \leftarrow \left(\bigotimes_{x \in r \cdot \langle a_h | y} b[x]\right) \,\&\, \mathsf{t}(y)$

end for

$b[r \cdot \langle a_h | m] \leftarrow 0$

$m \leftarrow a \triangleright (r \cdot | a_h \rangle m)$

end while

$b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}

end function

Decompositions into tuple preferences (example run)

function $\textsc{Decomp\_Tuple}(a, r)$

$a_h \leftarrow a \sqcap \overline{a^2}$

$b[r] \leftarrow 0$

$m \leftarrow a \triangleright r$

while $m \neq 0$ **do**

for all $y \in m$ **do**

$b[y] \leftarrow \left(\bigotimes_{x \in r \cdot \langle a_h | y} b[x]\right) \,\&\, \mathsf{t}(y)$

end for

$b[r \cdot \langle a_h | m] \leftarrow 0$

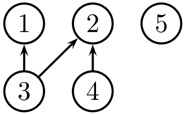
$m \leftarrow a \triangleright (r \cdot | a_h \rangle m)$

end while

$b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}

end function

given preference a

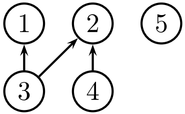


var	value
y	?
$\langle a_h \| y$	?
m	?
$\langle a_h \| m$	?
$b[x_1]$	?
$b[x_2]$	?
$b[x_3]$	?
$b[x_4]$	?
$b[x_5]$	?

Decompositions into tuple preferences (example run)

function DECOMP_TUPLE(a, r)
    $a_h \leftarrow a \sqcap \overline{a^2}$
    $b[r] \leftarrow 0$
    $m \leftarrow a \rhd r$
    while $m \neq 0$ **do**
        for all $y \in m$ **do**
            $b[y] \leftarrow \left(\bigotimes_{x \in r \cdot \langle a_h | y} b[x]\right) \,\&\, \mathsf{t}(y)$
        end for
        $b[r \cdot \langle a_h | m] \leftarrow 0$
        $m \leftarrow a \rhd (r \cdot |a_h\rangle m)$
    end while
    $b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}
end function

given preference a

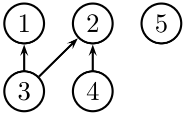


var	value
y	?
$\langle a_h \| y$	?
m	?
$\langle a_h \| m$	?
$b[x_1]$	?
$b[x_2]$	?
$b[x_3]$	?
$b[x_4]$	?
$b[x_5]$	?

Decompositions into tuple preferences (example run)

function DECOMP_TUPLE(a, r)
  $a_h \leftarrow a \sqcap \overline{a^2}$
  $b[r] \leftarrow 0$
  $m \leftarrow a \rhd r$
  while $m \neq 0$ **do**
    for all $y \in m$ **do**
      $b[y] \leftarrow \left(\bigotimes_{x \in r \cdot \langle a_h | y} b[x]\right) \,\&\, \mathsf{t}(y)$
    end for
    $b[r \cdot \langle a_h | m] \leftarrow 0$
    $m \leftarrow a \rhd (r \cdot | a_h \rangle m)$
  end while
  $b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}
end function

given preference a



var	value
y	?
$\langle a_h \| y$	?
m	?
$\langle a_h \| m$	?
$b[x_1]$	?
$b[x_2]$	?
$b[x_3]$	?
$b[x_4]$	?
$b[x_5]$	?

Decompositions into tuple preferences (example run)

function $\textsc{Decomp\_Tuple}(a, r)$

  $a_h \leftarrow a \sqcap \overline{a^2}$

  $b[r] \leftarrow 0$

  $m \leftarrow a \triangleright r$

  while $m \neq 0$ **do**

    for all $y \in m$ **do**

      $b[y] \leftarrow \left(\bigotimes_{x \in r \cdot \langle a_h | y} b[x]\right) \,\&\, \mathsf{t}(y)$

    end for

    $b[r \cdot \langle a_h | m] \leftarrow 0$

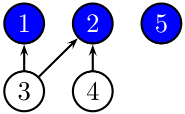
    $m \leftarrow a \triangleright (r \cdot |a_h\rangle m)$

  end while

  $b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}

end function

given preference a



var	value
y	?
$\langle a_h \vert y$	?
m	?
$\langle a_h \vert m$	?
$b[x_1]$	0
$b[x_2]$	0
$b[x_3]$	0
$b[x_4]$	0
$b[x_5]$	0

Decompositions into tuple preferences (example run)

function $\textsc{Decomp\_Tuple}(a, r)$

  $a_h \leftarrow a \sqcap \overline{a^2}$

  $b[r] \leftarrow 0$

  $m \leftarrow a \rhd r$

  while $m \neq 0$ **do**

    for all $y \in m$ **do**

      $b[y] \leftarrow \left(\bigotimes_{x \in r \cdot \langle a_h | y} b[x]\right) \,\&\, \mathsf{t}(y)$

    end for

    $b[r \cdot \langle a_h | m] \leftarrow 0$

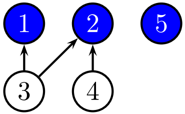
    $m \leftarrow a \rhd (r \cdot |a_h\rangle m)$

  end while

  $b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}

end function

given preference a

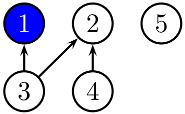


var	value
y	?
$\langle a_h \vert y$	?
m	$x_1 + x_2 + x_5$
$\langle a_h \vert m$	0
$b[x_1]$	0
$b[x_2]$	0
$b[x_3]$	0
$b[x_4]$	0
$b[x_5]$	0

Decompositions into tuple preferences (example run)

function $\textsc{Decomp\_Tuple}(a, r)$
 $a_h \leftarrow a \sqcap \overline{a^2}$
 $b[r] \leftarrow 0$
 $m \leftarrow a \triangleright r$
 while $m \neq 0$ **do**
  for all $y \in m$ **do**
   $b[y] \leftarrow \left(\bigotimes_{x \in r \cdot \langle a_h | y} b[x]\right) \,\&\, \mathsf{t}(y)$
  end for
  $b[r \cdot \langle a_h | m] \leftarrow 0$
  $m \leftarrow a \triangleright (r \cdot | a_h \rangle m)$
 end while
 $b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}
end function

given preference a

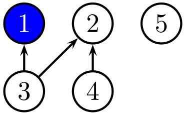


var	value
y	?
$\langle a_h \| y$	?
m	$x_1 + x_2 + x_5$
$\langle a_h \| m$	0
$b[x_1]$	0
$b[x_2]$	0
$b[x_3]$	0
$b[x_4]$	0
$b[x_5]$	0

Decompositions into tuple preferences (example run)

function $\textsc{Decomp\_Tuple}(a, r)$
$a_h \leftarrow a \sqcap \overline{a^2}$
$b[r] \leftarrow 0$
$m \leftarrow a \rhd r$
while $m \neq 0$ **do**
for all $y \in m$ **do**
$b[y] \leftarrow \left(\bigotimes_{x \in r \cdot \langle a_h | y} b[x]\right) \,\&\, \mathsf{t}(y)$
end for
$b[r \cdot \langle a_h | m] \leftarrow 0$
$m \leftarrow a \rhd (r \cdot |a_h\rangle m)$
end while
$b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}
end function

given preference a



var	value
y	x_1
$\langle a_h \| y$	0
m	$x_1 + x_2 + x_5$
$\langle a_h \| m$	0
$b[x_1]$	0
$b[x_2]$	0
$b[x_3]$	0
$b[x_4]$	0
$b[x_5]$	0

Decompositions into tuple preferences (example run)

function $\textsc{Decomp\_Tuple}(a, r)$

$a_h \leftarrow a \sqcap \overline{a^2}$

$b[r] \leftarrow 0$

$m \leftarrow a \rhd r$

while $m \neq 0$ **do**

for all $y \in m$ **do**

$b[y] \leftarrow \left(\bigotimes_{x \in r \cdot \langle a_h | y} b[x]\right) \,\&\, \mathsf{t}(y)$

end for

$b[r \cdot \langle a_h | m] \leftarrow 0$

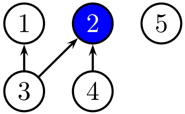
$m \leftarrow a \rhd (r \cdot |a_h\rangle m)$

end while

$b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}

end function

given preference a

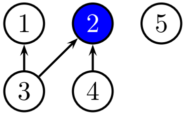


var	value
y	x_1
$\langle a_h \vert y$	0
m	$x_1 + x_2 + x_5$
$\langle a_h \vert m$	0
$b[x_1]$	$\mathsf{t}(x_1)$
$b[x_2]$	0
$b[x_3]$	0
$b[x_4]$	0
$b[x_5]$	0

Decompositions into tuple preferences (example run)

function DECOMP_TUPLE(a, r)
  $a_h \leftarrow a \sqcap \overline{a^2}$
  $b[r] \leftarrow 0$
  $m \leftarrow a \rhd r$
  while $m \neq 0$ **do**
    for all $y \in m$ **do**
      $b[y] \leftarrow \left(\bigotimes_{x \in r \cdot \langle a_h | y} b[x]\right) \,\&\, \mathsf{t}(y)$
    end for
    $b[r \cdot \langle a_h | m] \leftarrow 0$
    $m \leftarrow a \rhd (r \cdot |a_h\rangle m)$
  end while
  $b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}
end function

given preference a

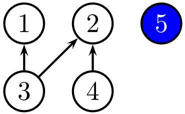


var	value
y	x_2
$\langle a_h \| y$	0
m	$x_1 + x_2 + x_5$
$\langle a_h \| m$	0
$b[x_1]$	$\mathsf{t}(x_1)$
$b[x_2]$	0
$b[x_3]$	0
$b[x_4]$	0
$b[x_5]$	0

Decompositions into tuple preferences (example run)

function DECOMP_TUPLE(a, r)
  $a_h \leftarrow a \sqcap \overline{a^2}$
  $b[r] \leftarrow 0$
  $m \leftarrow a \triangleright r$
  while $m \neq 0$ **do**
    for all $y \in m$ **do**
      $b[y] \leftarrow \left(\bigotimes_{x \in r \cdot \langle a_h | y} b[x]\right) \,\&\, \mathtt{t}(y)$
    end for
    $b[r \cdot \langle a_h | m] \leftarrow 0$
    $m \leftarrow a \triangleright (r \cdot |a_h\rangle m)$
  end while
  $b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}
end function

given preference a

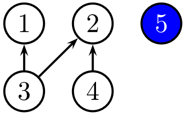


var	value
y	x_2
$\langle a_h \| y$	0
m	$x_1 + x_2 + x_5$
$\langle a_h \| m$	0
$b[x_1]$	$\mathtt{t}(x_1)$
$b[x_2]$	$\mathtt{t}(x_2)$
$b[x_3]$	0
$b[x_4]$	0
$b[x_5]$	0

Decompositions into tuple preferences (example run)

function $\textsc{Decomp\_Tuple}(a, r)$
  $a_h \leftarrow a \sqcap \overline{a^2}$
  $b[r] \leftarrow 0$
  $m \leftarrow a \rhd r$
  while $m \neq 0$ **do**
    for all $y \in m$ **do**
      $b[y] \leftarrow \left(\bigotimes_{x \in r \cdot \langle a_h | y} b[x]\right) \,\&\, \mathsf{t}(y)$
    end for
    $b[r \cdot \langle a_h | m] \leftarrow 0$
    $m \leftarrow a \rhd (r \cdot |a_h\rangle m)$
  end while
  $b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}
end function

given preference a



var	value
y	x_5
$\langle a_h\|y$	0
m	$x_1 + x_2 + x_5$
$\langle a_h\|m$	0
$b[x_1]$	$\mathsf{t}(x_1)$
$b[x_2]$	$\mathsf{t}(x_2)$
$b[x_3]$	0
$b[x_4]$	0
$b[x_5]$	0

Decompositions into tuple preferences (example run)

function $\textsc{Decomp\_Tuple}(a, r)$

$a_h \leftarrow a \sqcap \overline{a^2}$

$b[r] \leftarrow 0$

$m \leftarrow a \rhd r$

while $m \neq 0$ **do**

for all $y \in m$ **do**

$b[y] \leftarrow \left(\bigotimes_{x \in r \cdot \langle a_h | y} b[x]\right) \,\&\, \mathtt{t}(y)$

end for

$b[r \cdot \langle a_h | m] \leftarrow 0$

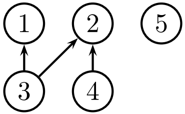
$m \leftarrow a \rhd (r \cdot |a_h\rangle m)$

end while

$b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}

end function

given preference a

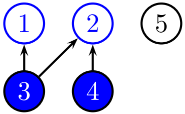


var	value
y	?
$\langle a_h \| y$	?
m	$x_1 + x_2 + x_5$
$\langle a_h \| m$	0
$b[x_1]$	$\mathtt{t}(x_1)$
$b[x_2]$	$\mathtt{t}(x_2)$
$b[x_3]$	0
$b[x_4]$	0
$b[x_5]$	$\mathtt{t}(x_5)$

Decompositions into tuple preferences (example run)

function $\textsc{Decomp\_Tuple}(a, r)$
  $a_h \leftarrow a \sqcap \overline{a^2}$
  $b[r] \leftarrow 0$
  $m \leftarrow a \triangleright r$
  while $m \neq 0$ **do**
    for all $y \in m$ **do**
      $b[y] \leftarrow \left(\bigotimes_{x \in r \cdot \langle a_h | y} b[x]\right) \,\&\, \mathsf{t}(y)$
    end for
    $b[r \cdot \langle a_h | m] \leftarrow 0$
    $m \leftarrow a \triangleright (r \cdot | a_h \rangle m)$
  end while
  $b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}
end function

given preference a

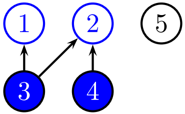


var	value
y	?
$\langle a_h \| y$	?
m	$x_1 + x_2 + x_5$
$\langle a_h \| m$	0
$b[x_1]$	$\mathsf{t}(x_1)$
$b[x_2]$	$\mathsf{t}(x_2)$
$b[x_3]$	0
$b[x_4]$	0
$b[x_5]$	$\mathsf{t}(x_5)$

Decompositions into tuple preferences (example run)

function $\text{DECOMP\_TUPLE}(a, r)$
  $a_h \leftarrow a \sqcap \overline{a^2}$
  $b[r] \leftarrow 0$
  $m \leftarrow a \triangleright r$
  while $m \neq 0$ **do**
    for all $y \in m$ **do**
      $b[y] \leftarrow \left(\bigotimes_{x \in r \cdot \langle a_h | y} b[x]\right) \,\&\, \mathsf{t}(y)$
    end for
    $b[r \cdot \langle a_h | m] \leftarrow 0$
    $m \leftarrow a \triangleright (r \cdot |a_h\rangle m)$
  end while
  $b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}
end function

given preference a

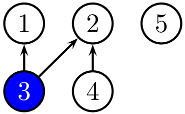


var	value
y	?
$\langle a_h \| y$	?
m	$x_3 + x_4$
$\langle a_h \| m$	$x_1 + x_2$
$b[x_1]$	$\mathsf{t}(x_1)$
$b[x_2]$	$\mathsf{t}(x_2)$
$b[x_3]$	0
$b[x_4]$	0
$b[x_5]$	$\mathsf{t}(x_5)$

Decompositions into tuple preferences (example run)

function DECOMP_TUPLE(a, r)
  $a_h \leftarrow a \sqcap \overline{a^2}$
  $b[r] \leftarrow 0$
  $m \leftarrow a \triangleright r$
  while $m \neq 0$ **do**
    for all $y \in m$ **do**
      $b[y] \leftarrow \left(\bigotimes_{x \in r \cdot \langle a_h | y} b[x]\right) \,\&\, \mathsf{t}(y)$
    end for
    $b[r \cdot \langle a_h | m] \leftarrow 0$
    $m \leftarrow a \triangleright (r \cdot |a_h\rangle m)$
  end while
  $b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}
end function

given preference a



var	value
y	?
$\langle a_h \| y$	?
m	$x_3 + x_4$
$\langle a_h \| m$	$x_1 + x_2$
$b[x_1]$	$\mathsf{t}(x_1)$
$b[x_2]$	$\mathsf{t}(x_2)$
$b[x_3]$	0
$b[x_4]$	0
$b[x_5]$	$\mathsf{t}(x_5)$

Decompositions into tuple preferences (example run)

function $\textsc{Decomp\_Tuple}(a, r)$

$a_h \leftarrow a \sqcap \overline{a^2}$

$b[r] \leftarrow 0$

$m \leftarrow a \rhd r$

while $m \neq 0$ **do**

for all $y \in m$ **do**

$b[y] \leftarrow \left(\bigotimes_{x \in r \cdot \langle a_h | y} b[x]\right) \,\&\, \mathsf{t}(y)$

end for

$b[r \cdot \langle a_h | m] \leftarrow 0$

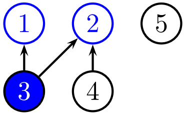
$m \leftarrow a \rhd (r \cdot |a_h\rangle m)$

end while

$b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}

end function

given preference a



var	value
y	x_3
$\langle a_h \| y$	$x_1 + x_2$
m	$x_3 + x_4$
$\langle a_h \| m$	$x_1 + x_2$
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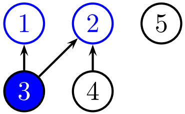
$b[x_5]$	$\mathsf{t}(x_5)$

Decompositions into tuple preferences (example run)

function $\textsc{Decomp\_Tuple}(a, r)$
- $a_h \leftarrow a \sqcap \overline{a^2}$
- $b[r] \leftarrow 0$
- $m \leftarrow a \triangleright r$
- **while** $m \neq 0$ **do**
  - **for all** $y \in m$ **do**
    - $b[y] \leftarrow \left(\bigotimes_{x \in r \cdot \langle a_h | y} b[x]\right) \,\&\, \mathsf{t}(y)$
  - **end for**
  - $b[r \cdot \langle a_h | m] \leftarrow 0$
  - $m \leftarrow a \triangleright (r \cdot |a_h\rangle m)$
- **end while**
- $b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}

end function

given preference a



var	value
y	x_3
$\langle a_h \| y$	$x_1 + x_2$
m	$x_3 + x_4$
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$b[x_4]$	0
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$b[r \cdot \langle a_h | m] \leftarrow 0$

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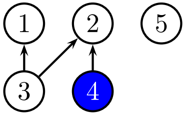
given preference a



var	value
y	
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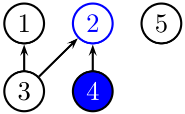
given preference a



var	value
y	x_4
$\langle a_h \| y$	x_2
m	$x_3 + x_4$
$\langle a_h \| m$	$x_1 + x_2$
$b[x_1]$	$\mathsf{t}(x_1)$
$b[x_2]$	$\mathsf{t}(x_2)$
$b[x_3]$	$(\mathsf{t}(x_1) \otimes \mathsf{t}(x_2)) \& \mathsf{t}(x_3)$
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end for

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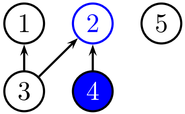
end while

$b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}

end function

given preference a



var	value
y	x_4
$\langle a_h \| y$	x_2
m	$x_3 + x_4$
$\langle a_h \| m$	$x_1 + x_2$
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$b[x_2]$	$\mathsf{t}(x_2)$
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end for
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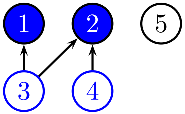

var	value
y	?
$\langle a_h \vert y$	?
m	$x_3 + x_4$
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$b[x_1]$	$\mathsf{t}(x_1)$
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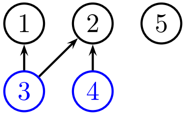
    end for
    $b[r \cdot \langle a_h | m] \leftarrow 0$
    $m \leftarrow a \triangleright (r \cdot | a_h \rangle m)$
  end while
  $b_{\text{res}} \leftarrow \bigotimes_{x \in r} b[x]$; **return** b_{res}
end function

given preference a



var	value
y	?
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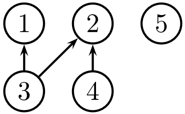
given preference a



var	value
y	?
$\langle a_h \vert y$	?
m	0
$\langle a_h \vert m$	0
$b[x_1]$	0
$b[x_2]$	0
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$b[r \cdot \langle a_h | m] \leftarrow 0$

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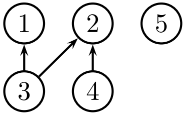

var	value
y	?
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$b[x_1]$	0
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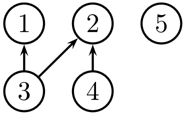
end function

Final result:

$b_{\text{res}} = b[x_3] \otimes b[x_4] \otimes b[x_5]$

given preference a



var	value
y	?
$\langle a_h \| y$	?
m	0
$\langle a_h \| m$	0
$b[x_1]$	0
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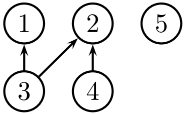
given preference a



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y	?
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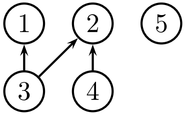

Final result:

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The idea of the optimization

- The decomposition creates a lot of redundancy
- Consider $b = \mathsf{t}(x_1 + x_2)$ on $r = x_1 + \ldots + x_5$
- The decomposition into tuple preferences and $\{\&, \otimes\}$ results in

$$(\mathsf{t}(x_1) \,\&\, (\mathsf{t}(x_3) \otimes \mathsf{t}(x_4) \otimes \mathsf{t}(x_5))) \otimes (\mathsf{t}(x_2) \,\&\, (\mathsf{t}(x_3) \otimes \mathsf{t}(x_4) \otimes \mathsf{t}(x_5)))$$

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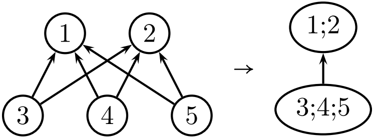
- Idea: merge “equivalent” nodes in the Hasse diagram



where $(i_1; \ldots; i_k)$ is short for $x_{i_1} + \ldots + x_{i_k}$.

Elimination of equivalent nodes

Definition (Minimized preference)

For a preference a and a data set r we define:

$$u \sim_{a,r} v \quad \Leftrightarrow_{df} \quad r \cdot |a\rangle u = r \cdot |a\rangle v \quad \wedge \quad r \cdot \langle a| u = r \cdot \langle a| v \ .$$

We define $r_{\min} =_{df} r/\!\sim_{a,r}$ containing equivalence classes $[\![x]\!]$.

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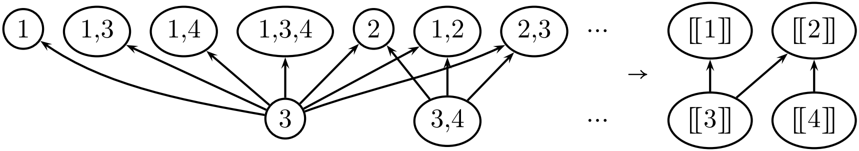
We define $r_{\min} =_{df} r/\sim_{a,r}$ containing equivalence classes $[\![x]\!]$.

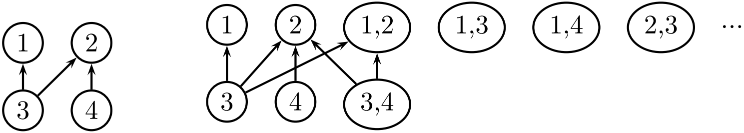
We apply this to the power set preference π_1^a, reducing the graph largely:



$\Rightarrow$ In this case the graph of $(\pi_1^a)_{\min}$ is isomorphic to that of a.

More examples (1/2)

The N-shaped preference and $\pi_2^{(\dots)}$:



1) a 2) π_2^a

More examples (1/2)

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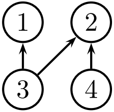
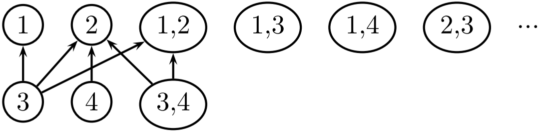
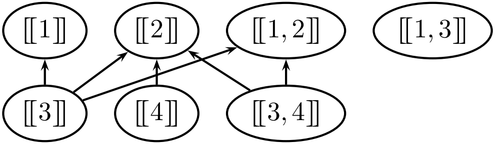


1) a



2) π_2^a

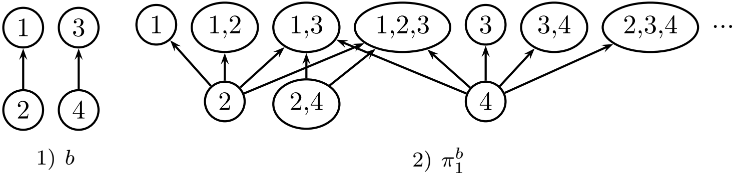


3) $(\pi_2^a)_{\min}$

⇒ The only equivalent nodes are the incomparable ones.

More examples (2/2)

We consider two parallel chains and $\pi_1^{(\dots)}$:



More examples (2/2)

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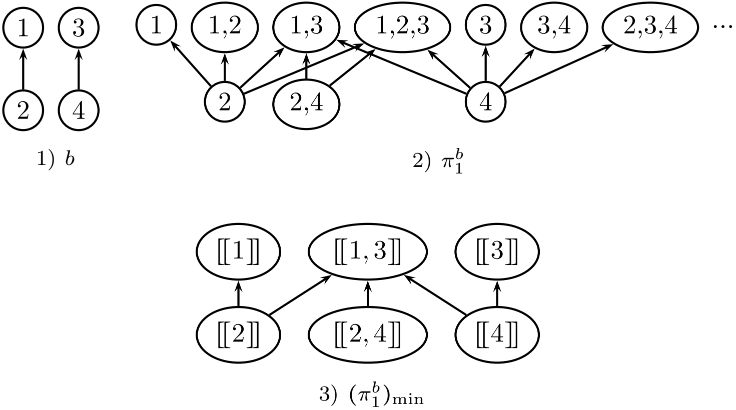

⇒ The complexity of the power set preference can be largely reduced

⇒ Graph of $(\pi_1^b)_{\min}$ is still more complex than the original preference b

Conclusion and outlook

What was done:

- The term complexity of the decompositions can be reduced
- This optimization is useful for decomposed power set preferences
- See the paper for an empirical study underlining that

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What was done:

- The term complexity of the decompositions can be reduced
- This optimization is useful for decomposed power set preferences
- See the paper for an empirical study underlining that
- Implementations of the algorithms are available at `http://www.p-roocks.de/powerset-prefs-ramics.r`

  based on our freely available CRAN R package rPref (`http://www.p-roocks.de/rpref`)

Conclusion and outlook

What was done:

- The term complexity of the decompositions can be reduced
- This optimization is useful for decomposed power set preferences
- See the paper for an empirical study underlining that
- Implementations of the algorithms are available at `http://www.p-roocks.de/powerset-prefs-ramics.r`
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Open questions:

- Is there a closed formula determining the term length of a decomposition of a (power set) preference?
- How can we retrieve provable *minimal decompositions*?