Roughness by Residuals

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Approximations

Rough sets (without elements) ... but with characteristic functions ... but with subidentities

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Introduction

At-least and at-most approximations

\Box and \diamond

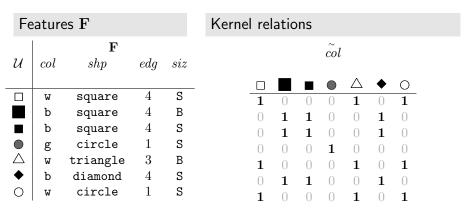
Morphology : $\varepsilon.(\cdot)$ and $\delta.(\cdot)$ w.r.t binary relations

KAT : $[\cdot] \cdot \text{ and } \langle \cdot | \cdot \text{ w.r.t tests}$

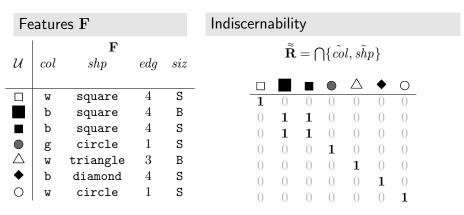
Residuals : wp and sp w.r.t p.o., preorder, ...

Approximations: $\llbracket \cdot \rrbracket \cdot$ and $\langle \cdot \rangle \cdot$ w.r.t equivalences

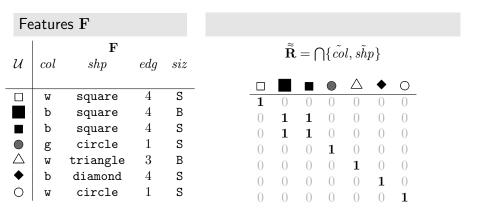
Features F						Attributes A											
			Α														
\mathcal{U}	col	ol shp edg siz			col			shp				edg			siz		
					W	g	b	с	t	d	s	1	3	4	S	В	
	W	square	4	S	1	0	0	0	0	0	1	0	0	1	1	0	
	b	square	4	В	0	0	1	0	0	0	1	0	0	1	0	1	
	b	square	4	S	0	0	1	0	0	0	1	0	0	1	1	0	
	g	circle	1	S	0	1	0	1	0	0	0	1	0	0	1	0	
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•	b	diamond	4	S	0	0	1	0	0	1	0	0	0	1	1	0	
0	W	circle	1	S	1	0	0	1	0	0	0	1	0	0	1	0	



<u>Def.</u> Kernel relation: $x \widetilde{f} y : \iff f(x) = f(y)$



Def.Kernel relation:
$$x \tilde{f} y :\iff f(x) = f(y)$$
Def.Indiscernability: $x \widetilde{\mathbf{R}} y :\iff x(\bigcap \mathbf{R}) y$



Goal: Find a smallest $\mathbf{R} \subseteq \operatorname{EquR}(\mathcal{U})$ that creates a finest $\tilde{\mathbf{R}}$! $(\tilde{\mathbf{R}} = 1 \text{ or } \tilde{\mathbf{R}} \approx T)$

Rough sets, "pointwise"

Def. Upper and Lower Approximations N.B.
$$R := \widetilde{\mathbb{R}}$$

$$\begin{bmatrix} R \end{bmatrix} s := \{x \in \mathcal{U} : [x]_R \subseteq s\}$$
(1)
$$\langle R \rangle s := \{x \in \mathcal{U} : [x]_R \cap s \neq \emptyset\}.$$
(2)

Iso-/Antitony of [[]] and $\langle\!\!|\;\rangle\!\!\rangle$ w.r.t. set and relation arguments

$$s \subseteq t \implies \begin{array}{ccc} \langle R \rangle s & \subseteq & \langle R \rangle t \\ \llbracket R \rrbracket s & \subseteq & \llbracket R \rrbracket t \end{array} \text{ but } P \subseteq R \implies \begin{array}{ccc} \langle P \rangle s & \subseteq & \langle R \rangle s \\ \llbracket P \rrbracket s & \supseteq & \llbracket R \rrbracket s \end{array}$$

Duality (as desired)

$$\llbracket R \rrbracket \overline{s} = \overline{\langle R \rangle s}. \tag{3}$$

We represent $s \subseteq \mathcal{U}$ by its characteristic relation $\dot{s} : \mathcal{U} \to \mathbf{2}$. We observe (pointwise):

$$x \in \llbracket R \rrbracket s \Longleftrightarrow [x]_R \subseteq s$$

Def. Upper and Lower Approximations, again.

$$\begin{bmatrix} R \end{bmatrix} s := \langle R \| \dot{s} | \{ \mathbf{1} \}$$

$$\langle R \| s := \langle R \| \overline{\dot{s}} | \{ \mathbf{0} \}$$

$$(4)$$

where

$$\begin{array}{rcl} \mathrm{RR}: & P \backslash\!\!\backslash Q & := & \overline{P^{\smile} \, \!\!\!\circ \, \overline{Q}} & \text{or} & R \subseteq P \backslash\!\!\backslash Q & \Longleftrightarrow & P \, \!\!\circ \, R \subseteq Q \\ \mathrm{LR}: & Q /\!\!/ P & := & \overline{\overline{Q} \, \!\!\circ \, P^{\smile}} & \text{or} & R \subseteq Q /\!\!/ P & \Longleftrightarrow & R \, \!\!\circ \, P \subseteq Q \end{array}$$

We represent $s \subseteq \mathcal{U}$ by its characteristic relation $\dot{s} : \mathcal{U} \to \mathbf{2}$. We observe (pointwise):

$$x \in [\![R]\!]s \Longleftrightarrow xRy \longrightarrow y \in s$$

Def. Upper and Lower Approximations, again.

$$\llbracket R \rrbracket s := \langle R \backslash \langle s | \{ 1 \}$$

$$\langle R \rangle s := \langle R \backslash \langle \bar{s} | \{ 0 \}$$

$$(4)$$

where

We represent $s \subseteq \mathcal{U}$ by its characteristic relation $\dot{s} : \mathcal{U} \to \mathbf{2}$. We observe (pointwise):

$$x \in \llbracket R \rrbracket s \Longleftrightarrow x \overline{R} y \lor y \dot{s} \mathbf{1}$$

Def. Upper and Lower Approximations, again.

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where

We represent $s \subseteq \mathcal{U}$ by its characteristic relation $\dot{s} : \mathcal{U} \to \mathbf{2}$. We observe (pointwise):

$$x \in \llbracket R \rrbracket s \Longleftrightarrow \neg (y \overline{\dot{s}} \mathbf{1} \land x R y)$$

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where

$$\begin{array}{rcl} \mathrm{RR}: & P \backslash\!\!\backslash Q & := & \underline{P^{\vee}}_{\$} \overline{Q} & \text{or} & R \subseteq P \backslash\!\!\backslash Q & \Longleftrightarrow & P_{\$} R \subseteq Q \\ \mathrm{LR}: & Q /\!\!/ P & := & \overline{Q}_{\$} P^{\vee} & \text{or} & R \subseteq Q /\!\!/ P & \Longleftrightarrow & R_{\$}^{\ast} P \subseteq Q \end{array}$$

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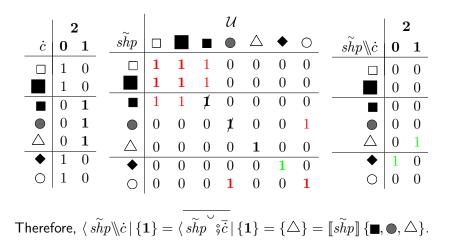
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Example



We represent $s \subseteq U$ by its subidentity $S = 1 \cap (s \times s)$ and define $S^- := 1 \cap \overline{S}$. We observe:

$$x \in \llbracket R \rrbracket s \Longleftrightarrow [x]_R \subseteq \mathsf{S}.\mathcal{U}$$

Def. Upper and Lower Approximations, again.

$$\llbracket R \rrbracket s := \operatorname{dom}(R \backslash S)$$

$$\langle R \rangle s := \operatorname{\overline{dom}}(R \backslash S^{-})$$

$$(6)$$

$$(7)$$

We represent $s \subseteq U$ by its subidentity $S = 1 \cap (s \times s)$ and define $S^- := 1 \cap \overline{S}$. We observe:

$$x \in \llbracket R \rrbracket s \Longleftrightarrow \forall y : xRy \longrightarrow y \mathsf{S}y$$

Def. Upper and Lower Approximations, again.

$$\llbracket R \rrbracket s := \operatorname{dom}(R \backslash S)$$

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$$(6)$$

$$(7)$$

$$\llbracket R \rrbracket \mathcal{U}/Q := \{\llbracket R \rrbracket c : c \in \mathfrak{c}\} = \{ \operatorname{dom}(R \backslash C) : c \in \mathfrak{c} \}$$
(8)
where $\mathfrak{c} = \mathcal{U}/Q$ is a target classification.

We represent $s \subseteq U$ by its subidentity $S = 1 \cap (s \times s)$ and define $S^- := 1 \cap \overline{S}$. We observe:

$$x \in \llbracket R \rrbracket s \Longleftrightarrow \forall y : x \overline{R} y \lor y \mathsf{S} y$$

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$$x \in \llbracket R \rrbracket s \Longleftrightarrow \forall y : \neg (y \overline{\mathsf{S}} y \land x R y)$$

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$$x \in [\![R]\!] s \Longleftrightarrow \forall y: \ x \, R^{\smile} \$ \overline{\mathsf{S}} \, y$$

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$$x \in \llbracket R \rrbracket s \Longleftrightarrow x \in R \backslash\!\!\backslash \mathsf{S} \mathcal{U}$$

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where $\mathfrak{c} = \mathcal{U}/Q$ is a target classification.

Rough sets by KAD

Preimages, domains and tests

With KAT we are given

$$\langle R | := \min \{ X \in \mathsf{U} : R \subseteq X \, {}^{\circ}_{\mathcal{F}} R \}$$
(9)

$$-\langle R| := \max\left\{X \in \mathsf{U} : X \, ; R \subseteq \overline{C}\right\} \tag{10}$$

Here, $C=\mathbb{T}$ (i.e. not a relative, but absolute complement). Then, by domain laws,

$$\langle R | \mathsf{S} = \langle R_{\mathsf{S}}^{\mathsf{s}} \mathsf{S} | = \langle \overline{R \backslash\!\!\backslash \mathsf{S}^{-}} | = \langle\!\!\langle R \rangle\!\!\rangle s \tag{11}$$

and, canonically,

$$[R|\mathsf{T} := -\langle R|\mathsf{T}^{-} = \langle R||\mathsf{T}| = [\![R]\!]t.$$
(12)

Rough sets by GC

... are for free.

With $[\![\cdot]\!] \;/\; \langle\!|\cdot\rangle\!\rangle$ being defined by $[\cdot|\cdot\;/\;\langle\cdot|\cdot,$ and reading sets as subidentities,

$$\langle R | s \subseteq t \iff \langle R | t^- \subseteq s^- \iff s \subseteq -\langle R | t^- \iff s \subseteq [R | t.$$
 (13)

What we get for free

•
$$\llbracket R \rrbracket \overline{s} = \overline{\langle R \rangle s}$$
 proving (3).

- $\llbracket R \rrbracket s = s \iff \langle R \rangle s = s$ proving (17, 18) in the paper.
- ▶ ...
- ▶ a nice metaphor: "RST is just a GC, with only equivalences".

Positive regions and refinement

How good is R to ...

- describe a classification \mathcal{U}/Q (or s/Q)...
- compared to a relation P?

Def. Positive regions

$$\llbracket R \lessdot P \rrbracket s/Q := \bigcup_{c \in s/Q} \llbracket R \lessdot P \rrbracket c := \bigcup_{c \in s/Q} \bigcup_{t \in c/P} \llbracket R \rrbracket t.$$
(14)

 $\llbracket R \lt P \rrbracket s/Q$ is the largest set of elements $x \in s$ s.t. $[x]_R \subseteq [x]_P \subseteq [x]_Q$.

Positive regions and refinement

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Def. Positive regions

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(14)

Positive regions by residuals

$$[R \lessdot P] s/Q = \langle (R \backslash P) \backslash Q | s = \operatorname{dom} ((R \backslash P) \backslash Q ; S)$$
(15)

s.t.
$$\llbracket R \rrbracket s = \operatorname{dom} \left(\left(R \backslash \hspace{-0.5mm} |\hspace{-0.5mm} \tilde{s} \rangle \rangle \hspace{-0.5mm} |\hspace{-0.5mm} S \rangle = \operatorname{dom} \left(R \backslash \hspace{-0.5mm} |\hspace{-0.5mm} S \rangle \right)$$

but
$$\bigcup \llbracket R \rrbracket (s/Q) = R \backslash \hspace{-0.5mm} |\hspace{-0.5mm} Q \hspace{-0.5mm} | \hspace{-0.5mm} s = \operatorname{dom} \left(R \backslash \hspace{-0.5mm} | \hspace{-0.5mm} Q \hspace{-0.5mm} | \hspace{-0.5mm} S \rangle \right)$$

R is better than P, if...

Def. Refinement

R refines P w.r.t H on s/Q, iff

$$R \succeq_{s/Q}^{H} P \iff [\![R \lt H]\!](s/Q) \supseteq [\![P \lt H]\!](s/Q)$$

$$\iff \operatorname{dom}\left(\left((R \backslash\!\!\backslash H) \backslash\!\!\backslash Q\right) \varsigma \mathsf{S}\right) \supseteq \operatorname{dom}\left(\left((P \backslash\!\!\backslash H) \backslash\!\!\backslash Q\right) \varsigma \mathsf{S}\right)$$
(16)

Def. R is a (*H*-relative) reduct of P (on s/Q), if...

1.
$$\mathbf{R} \subseteq \mathbf{P}$$
 (hence $\tilde{\mathbf{P}} \subseteq \tilde{\mathbf{R}}$ and $\mathbf{P} \succeq \mathbf{R}$)

2. R is a *largest* subset of P s.t. $\mathbf{R} \succeq \mathbf{P}$.

 $\operatorname{Red}(\mathbf{P})$ is the set of all reducts of \mathbf{P} ; $\operatorname{Cor}(\mathbf{P}) := \bigcap \operatorname{Red}(\mathbf{R})$.

R is better than P, if...

Def. Refinement

R refines P w.r.t H on s, iff

$$R \stackrel{H}{\succeq} P \quad :\iff \quad [\![R < H]\!] s \supseteq [\![P < H]\!] s \qquad (16)$$
$$\iff \quad \operatorname{dom}\left((R \backslash\!\!\backslash H); \mathsf{S}\right) \supseteq \operatorname{dom}\left((P \backslash\!\!\backslash H); \mathsf{S}\right)$$

Def. R is a (*H*-relative) reduct of P (on s/Q), if...

- 1. $\mathbf{R} \subseteq \mathbf{P}$ (hence $\tilde{\widetilde{\mathbf{P}}} \subseteq \tilde{\widetilde{\mathbf{R}}}$ and $\mathbf{P} \succeq \mathbf{R}$)
- 2. **R** is a *largest* subset of **P** s.t. $\mathbf{R} \succeq \mathbf{P}$.

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R is better than P, if...

Def. Refinement

R refines P w.r.t H on \mathcal{U} , iff

$$R \stackrel{H}{\succeq} P :\iff [\![R \lessdot H]\!] \mathcal{U} \supseteq [\![P \lessdot H]\!] \mathcal{U} \qquad (16)$$
$$\iff \operatorname{dom}(R \backslash\!\!\backslash H) \supseteq \operatorname{dom}(P \backslash\!\!\backslash H)$$

Def. R is a (*H*-relative) reduct of P (on s/Q), if...

1.
$$\mathbf{R} \subseteq \mathbf{P}$$
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 $\operatorname{Red}(\mathbf{P})$ is the set of all reducts of \mathbf{P} ; $\operatorname{Cor}(\mathbf{P}) \coloneqq \bigcap \operatorname{Red}(\mathbf{R})$.

We need to make a few points

It is nice to know that $\mathbf{R}\in \operatorname{Red}(\mathbf{P}),$ but the definition does not help finding such $\mathbf{R}.$

Def. "s" is an arbitrarily chosen, fixed element (point) of s.

Sets and Partitions

- ► Sets
 - characteristic relations: $\dot{s}: \mathcal{U} \to \mathbf{2}$ $s = \langle \dot{s} | 1$
 - subidentities: $S := 1 \cap (s \times s)$

 $s = \langle s | 1 \\ s = \langle \mathsf{S} | \mathcal{U}$

- Classifications
 - quotients s/R:
 - pointwise representation: $r \subseteq s$

$$\begin{split} s/R &= \{ \langle R | \, x : x \in s \} \\ s/R &= \{ [c] : c \in r \} \end{split}$$

Concrete Tasks

... sadly require concrete data (points)

Goal: Construct (efficiently) some ${\bf R}$ in ${\rm Red}({\bf P})$

Gedankenexperiment

Suppose, $R \in Cor(\mathbf{P})$.

- Then, $R \in \mathbf{R}$ for all $\mathbf{R} \in \operatorname{Red}(\mathbf{P})$.
- Hence, $\mathbf{R} \stackrel{\mathbf{P}}{\succ} \mathbf{Q}$ (strictly!).
- By definition, $\exists x : x \notin \llbracket (\mathbf{Q} \lessdot \mathbf{P} \rrbracket s \subset \llbracket \mathbf{R} \lessdot \mathbf{P} \rrbracket s \ni x.$
- $\blacktriangleright \text{ Then, } [x]_{\widetilde{\mathbf{R}}} \subseteq [x]_{\widetilde{\mathbf{P}}} \text{ but } [x]_{\mathbf{Q}} \not\subseteq [x]_{\widetilde{\mathbf{P}}}.$
- Therefore, there is a representation r of $s/\tilde{\mathbf{P}}$, s.t.

$$\exists r_1, r_2 : r_1 \overline{R} r_2 \wedge [r_1]_{\widetilde{\mathbf{Q}}} = [r_2]_{\widetilde{\mathbf{Q}}}$$
(17)

 $(\mathbf{Q} := \mathbf{R} - \{R\})$

Concrete Tasks

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 $(\mathbf{Q} := \mathbf{R} - \{R\})$

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- Therefore, there is a representation r of $s/\tilde{\mathbf{P}}$, s.t.

$$\exists \underline{r}_1, \underline{r}_2 : \underline{r}_1 \overline{R} \underline{r}_2 \wedge \underline{r}_1 \widetilde{\widetilde{\mathbf{Q}}} \underline{r}_2$$
(17)

Concrete Tasks

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Gedankenexperiment

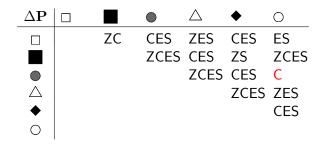
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- $\blacktriangleright \text{ Then, } [x]_{\widetilde{\mathbf{R}}} \subseteq [x]_{\widetilde{\mathbf{P}}} \text{ but } [x]_{\mathbf{Q}} \not\subseteq [x]_{\widetilde{\mathbf{P}}}.$
- Therefore, there is a representation r of $s/\tilde{\mathbf{P}}$, s.t. $\exists r_1, r_2 : r_1 \overline{R} r_2 \land \forall Q \in \mathbf{R} : Q \neq R \longrightarrow r_1 Q r_2$ (17)

A visual example

"Discernability matrices"



1. No entry is nil — hence, $\tilde{\mathbf{P}} = \bigcap \left\{ \tilde{siz}, \tilde{col}, \tilde{edg}, \tilde{shp} \right\} = 1$ 2. In any $\mathbf{R} \in \operatorname{Red}(\mathbf{P})$, \tilde{shp} is indispensable: it is essential 3. $\tilde{col} \in \operatorname{Cor}(\mathbf{P})$

Discernability of representatives

- ► If we have to make points, try making as few as possible.
- ► Speeding up exhaustive pointwise processes.

A (last) motivational example
$$\mathcal{U} = \{\Box, \blacksquare, \diamondsuit, \bigtriangleup, \bigtriangleup, \diamondsuit, \bigcirc, \bigcirc$$

1. Let $\mathbf{P} = \{col, s\tilde{h}p\}$.
2. Is col indispensable in \mathbf{P} w.r.t. 1?
2.1 Choose wisely $r_{\mathbf{P}-\{col\}} = r_{s\tilde{h}p} = \{\blacksquare, \blacklozenge, \bigcirc, \bigtriangleup\}$.
2.2 Consider \blacksquare : Then, $[\blacksquare]_{\widetilde{\mathbf{P}}} = \{\blacksquare\}$.
2.3 But: $\blacksquare \notin [\{s\tilde{h}p\} < \mathbf{P}]\mathcal{U}$
because $[\blacksquare]_{s\tilde{h}p} = \{\blacksquare, \Box\} \not\subseteq \{\blacksquare\} = [\blacksquare]_{\widetilde{\mathbf{P}}}$
3. Hence, col is essential

Finding Reducts (Skowron's exhaustive approach)

- 1. (Compute $\mathfrak{c} := s/Q$.)
- 2. (Compute $h := \llbracket \mathbf{R} \lessdot Q \rrbracket s.$)
- 4. For every $x, y \in \mathcal{U}$, compute $\{R : R \in \mathbf{R} \land x\overline{R}y\}$ 5. $\operatorname{Cor}_{\mathfrak{c}}(\mathbf{R}) := \{R : \exists x, y \in r : x \overline{\bigcap \mathbf{R} - \{R\}}y\}$ 6. For every $\mathbf{P} \subseteq \mathbf{R} - \operatorname{Cor}_{\mathfrak{c}}(\mathbf{R}) :$ $\mathbf{Q} := \mathbf{P} \cup \operatorname{Cor}_{\mathfrak{c}}(\mathbf{R})s$ is a Q-reduct of \mathbf{R} w.r.t. s, iff: $\mathbf{Q} \succeq_{s}^{Q} \mathbf{R}$ and \mathbf{Q} is not a superset of any other reduct.

Finding Reducts (A not so exhaustive approach)

- 1. (Compute $\mathfrak{c} := s/Q$.)
- 2. (Compute $h := \llbracket \mathbf{R} \lessdot Q \rrbracket s.$)
- 3. Guess a suitable $r := \{ c : c \in h/Q \}$.
- 4. For every $x, y \in \mathbf{r}$, compute $\{R : R \in \mathbf{R} \land x\overline{R}y\}$
- 5. $\operatorname{Cor}_{\mathfrak{c}}(\mathbf{R}) := \left\{ R : \exists x, y \in r : x \overline{\bigcap \mathbf{R} \{R\}} y \right\}$
- 6. For every $\mathbf{P} \subseteq \mathbf{R} \operatorname{Cor}_{\mathfrak{c}}(\mathbf{R})$ ordered by voodoo:
 - $\mathbf{Q} := \mathbf{P} \cup \operatorname{Cor}_{\mathfrak{c}}(\mathbf{R})s \text{ is a } Q \text{-reduct of } \mathbf{R} \text{ w.r.t. } s, \text{ iff:} \\ \mathbf{Q} \succeq_{s}^{Q} \mathbf{R} \text{ and } \mathbf{Q} \text{ is not a superset of any other reduct.}$

Conclusion

Summary

- Rough sets by residuals
- ► Rough sets by GC
- An algorithm for reduct construction

Prospects

- Rough sets by KAT (if not done yet)
- Rough sets by morphology (if not done yet)
- ► Rough sets by formal concept analysis (if not done yet)
- … automatically create a searchable space of logic programs with ⊤ being a program s.t. P ∀ E⁻ and ⊥ s.t. Q ⊢ E⁺ with ≈ as p.o. (not done yet)

 $you \in dom (Thanks; ATTENTION).$