Formal Methods II

4th Year Course — LMCC (7008N2) + LESI (5308P3)
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Introduction

The main theme of this project is the specification and (partial) calculation of a software package which monitors the activity of a particular kind of organization. Part of this specification is provided as a starting point. Additional information is provided in the form of WWW links to lecture notes or other projects which, if carefully studied, will make the project's overall task considerably simpler.

The structure of this document is as follows: section **Starting Specification Model** describes the problem domain while drafting a formal specification of a collection of basic datatypes which is put forward as a starting point. Section **Project Proposal** contains the actual requirements of the project.

Starting Specification Model

The kind of organization we have in mind is structured in departments. Departments may be organized hierarchically in sub-departments and so son. So our model will start by the following datatype

$$OrgSt \quad \cong \quad DepId \rightharpoonup Dept \qquad \text{/* } OrgSt \text{ = organization structure */}$$

where $DepId$ (= department identifier) should be regarded atomic (*e.g.* alphanumeric) and

$$Dept \quad \cong \quad DepInf + OrgSt \qquad \text{/* } Dept \text{ = department */}$$

where $DepInf$ will record each department (and sub-department) information, most of the details of which (*e.g.* director, employees, address *etc.*) are ignored for the sake of simplicity. Instead, we will focus on the *tasks* which a particular (sub)department accomplishes within the organization and $DepInf$ will record, for each task in which the (sub)department collaborates, the amount of resources (*e.g.* manpower, equipment, materials *etc.*) needed per unit of time (*e.g.* week):

$$\begin{array}{rcll} DepInf & \cong & TaskId \rightharpoonup Reso & \text{/* } TaskId \text{ = task identifier */} \\ Reso & \cong & ResId \rightharpoonup \mathbb{N} & \text{/* } ResId \text{ = resource identifier */} \end{array}$$

Next, one needs to consider how tasks are performed by the organization. Each task will have a unique identifier ($TaskId$) and some associated information ($TaskInf$). This includes a textual description ($Desc$), how long it takes ($Span$) and which other tasks ($Preds$) need to be completed before it can be triggered:

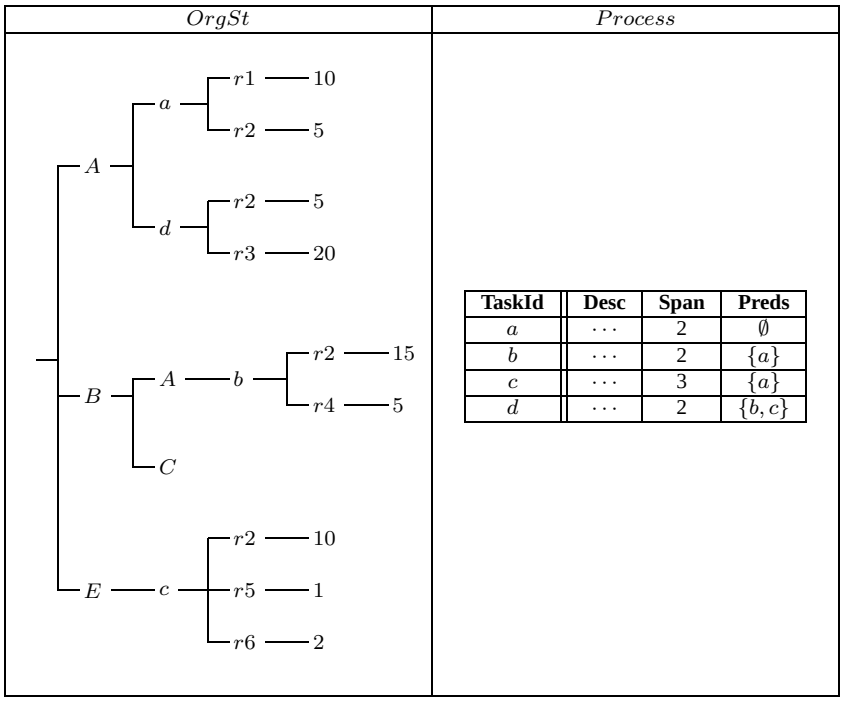
$$\begin{array}{rcll} Process & \cong & TaskId \rightharpoonup TaskInf & \text{/* } Process \text{ = tasks' network */} \\ TaskInf & \cong & Desc \times Span \times Preds & \text{/* } TaskInf \text{ = task information */} \\ Desc & \cong & Text & \text{/* } Desc \text{ = task description */} \\ Span & \cong & \mathbb{N} & \text{/* } Span \text{ = task duration */} \\ Preds & \cong & 2^{TaskId} & \text{/* } Preds \text{ = predecessor tasks */} \end{array}$$

The model of datatype $Text$ will be left unspecified since no functionality will interfere with it (any suitable model of textual data will do).

Putting everything together, we obtain a model of what we understand about an organization on the whole:

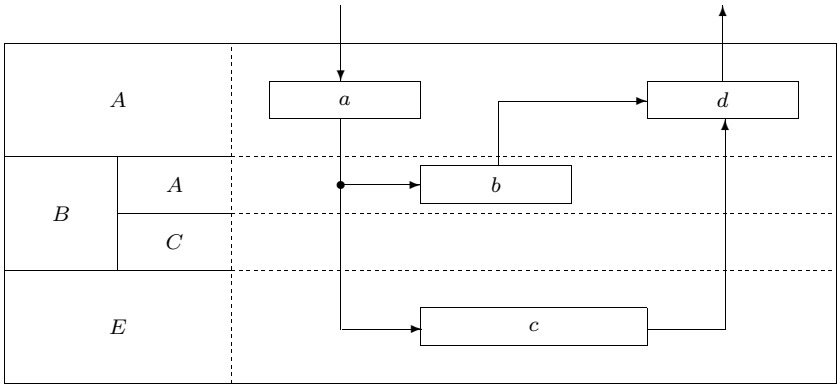
$$Org \quad \cong \quad OrgSt \times Process \qquad \text{/* } Org \text{ = overall organization */}$$

For instance, the following table depicts a particular ("toy") organization. In the $OrgSt$ part, capital letters represent $DepId$ values, lower case letters represent $TaskId$ values and other non-numeric symbols represent $ResId$ values. In the $Process$ part, the data-field $Desc$ is not represented:



TaskId	Desc	Span	Preds
a	$\cdots$	2	$\emptyset$
b	$\cdots$	2	$\{a\}$
c	$\cdots$	3	$\{a\}$
d	$\cdots$	2	$\{b, c\}$

As we shall see shortly, the functionality of this model will be targetted at specifying what we mean by a valid task schedule, that is, one which does not violate task precedence. There will be — or so we expect — a "best schedule", that is, one in which tasks are activated as early as possible. For instance, the following diagram pictures such a schedule concerning the "toy" organization above, where task box width is proportional to span (so, it is as if a time axis is considered on the horizontal direction):



A diagram of this kind will also be called a *process* schedule. It is clear from this diagram that sub-department C of department B does nothing in this process. However, it may participate in another process. Because our model does not allow the same organization to perform more than a single process, we have better enhance it in this direction:

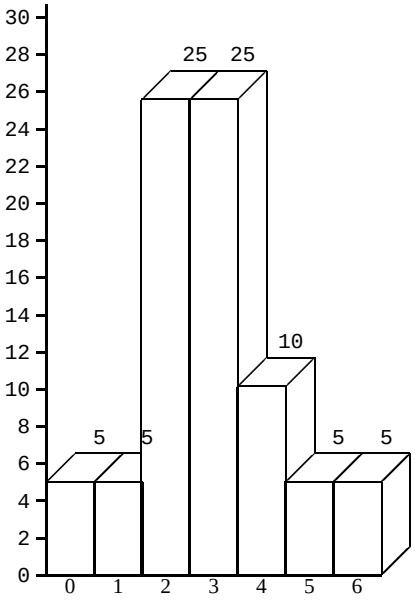
$$Org \quad \cong \quad OrgSt \times Processes \qquad \text{/* New version of } Org \text{ */} \tag{1}$$

where

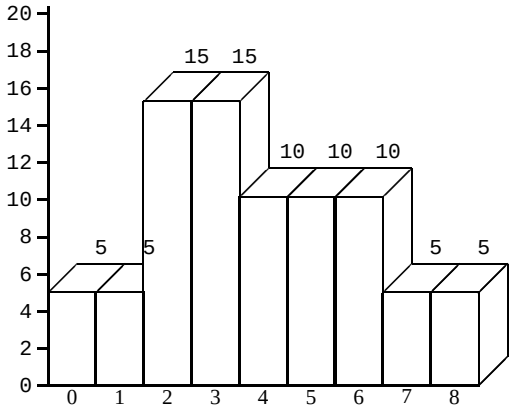
$$Processes \quad \cong \quad ProId \rightharpoonup Process \qquad \text{/* } ProId \text{ = process identifier */} \tag{2}$$

Another aspect which will concern us has to do with resource management. Resource consumption will depend upon task scheduling. For instance, the following diagram illustrates the overall evolution of resource $r2$ for the organization and particular schedule depicted above [1]:

[1] Every $0 \leq i \leq 6$ in the x-axis represents time span $[i, i+1[$.



It may be useful to adopt a schedule other than the "best" if, up to a predefined time limit (deadline), this will spread tasks along the time axis so as to render resource consumption as even as possible. For instance, should it be acceptable to shift tasks c and d later by 2 units of span, so that b and c do not overlap, than the histogram of resource $r2$ would be more even, as follows:



Part of the overall task of this project consists of equipping the datatypes just presented (or any elaboration of these) with functionality capable of express the above operations. Later on, part of the obtained model will be subject to a calculation process leading to a relational implementation.

Project Proposal

The model underlying this project is an evolution of the $ActPlan$ and Orc data models which can be found in [Oli97] as exercises [2]. The following requirements should be taken into account in your project report:

1. Specify a datatype invariant

$$\text{inv-}Process \quad : \quad Process \longrightarrow 2$$

ensuring not only that all tasks which precede some given task are are known tasks, but also acyclicness of the task precedence graph.

[2] See, for instance, Exercise 2.62 (p.127) and Exercise 2.70 (p.133).

2. Specify datatypes $Time$ and $Schedule$ and, on top of these and of the other datatypes already specified in section **Starting Specification Model**, provide a formal specification of (at least) four of the following six functions, defining preconditions and/or auxiliary functions wherever convenient:

$$
\begin{array}{rcl}
bestSchedule & : & Process \longrightarrow Schedule \\
bestSchedule(db) & \stackrel{\text{def}}{=} & \ldots \text{ /* This function should compute, for each task, its earliest possible start, which is dependent upon the start of its antecedents and so on. Invariant inv-}Process\text{ is assumed. */}
\end{array}
$$

$$
\begin{array}{rcl}
checkSchedule1 & : & Schedule \times Process \longrightarrow 2 \\
checkSchedule1(s,p) & \stackrel{\text{def}}{=} & \ldots \text{ /* This function checks the validity of schedule } s \text{ relative to the task precedences of process } p. \text{ */}
\end{array}
$$

$$
\begin{array}{rcl}
checkSchedule2 & : & Schedule \times Process \times Time \longrightarrow 2 \\
checkSchedule2(s,p,t) & \stackrel{\text{def}}{=} & \ldots \text{ /* This function checks whether schedule } s \text{ for process } p \text{ meets deadline } t. \text{ */}
\end{array}
$$

$$
\begin{array}{rcl}
shiftSchedule & : & Schedule \times Process \times (TaskId \rightharpoonup \mathbb{N}) \longrightarrow Schedule \\
shiftSchedule(s,p,m) & \stackrel{\text{def}}{=} & \ldots \text{ /* This function gets schedule } s \text{ relative to process } p \text{ and shifts all tasks in } m \text{ by the relevant time delays. The outcome should be the best schedule which accommodates such delays. */}
\end{array}
$$

$$
\begin{array}{rcl}
getDepts & : & OrgSt \longrightarrow DepId^{\star} \\
getDepts(o) & \stackrel{\text{def}}{=} & \ldots \text{ /* This function lists the “paths” which describe all departments of a given organization } o. \text{ */}
\end{array}
$$

$$
\begin{array}{rcl}
genHistogram & : & Org \times ProdId \times Schedule \times DepId^{\star} \times ResId \longrightarrow (Time \rightharpoonup \mathbb{N}) \\
genHistogram(o,p,s,d,r) & \stackrel{\text{def}}{=} & \ldots \text{ /* Let } o \text{ be an organization, } p \text{ be one of its processes, } s \text{ be a valid schedule for } p, \text{ } d \text{ be a department of } o \text{ and } r \text{ be a resource of } d. \text{ This function should compute the evolution in time of the consumption by department } d \text{ of resource } r \text{ in process } p, \text{ as implied by schedule } s. \text{ */}
\end{array}
$$

3. Build a VDM++ /VDM-SL prototype for the whole project structured such as the `stack` example available from

   `http://www.di.uminho.pt/~jno/tgz/mii0203mp.zip`.

4. Build a graphical user interface for your prototype (see the VDMTOOLS © documentation for details how to interface the tools with other technologies).
5. Calculate the relational model implementation of the Org datatype. Record the abstraction invariant (AI) relative to each step.
6. Describe the outcome of your calculations in a database language or diagrammatic notation, *e.g.* UML ou Entity-relationship diagrams. (See *e.g.* section 8.9 of [Oli97].)
7. Chose one of the operations you have specified above or invent a new one, and calculate its implementation at relational level.

References

[Oli97] J. N. Oliveira. *Métodos Formais de Programação*. University of Minho, 4^{th} edition, 1997. Textbook (489 p. in Portuguese [3]).

[3] zipped 2.8 Mb (inc. PDF and PS) file available from `http://www.di.uminho.pt/~jno/tgz/mfp.zip`.