

Leis do Cálculo Relacional

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AXIOMS: \cdot, \subseteq

(associativity)	$(R \cdot S) \cdot T = R \cdot (S \cdot T)$
(identity)	$R = R \cdot id = id \cdot R$
(\subseteq reflexivity)	$R \subseteq R$
(\subseteq transitivity)	$R \subseteq S \wedge S \subseteq T \Rightarrow R \subseteq T$
(\subseteq anti-symmetry)	$R \subseteq S \wedge S \subseteq R \Rightarrow R = S$
(monotonicity)	$S \subseteq T \wedge R \subseteq U \Rightarrow S \cdot R \subseteq T \cdot U$

AXIOMS: $=$

(ping-pong)	$R = S \equiv R \subseteq S \wedge S \subseteq R$
(indirection)	$R = S \equiv \forall X \cdot (X \subseteq R \wedge X \subseteq S)$

AXIOMS: \cap, \circ

(\cap universal)	$X \subseteq (R \cap S) \equiv X \subseteq R \wedge X \subseteq S$
(\circ involution)	$R^{\circ\circ} = R$
(\circ monotonicity)	$R \subseteq S \equiv R^{\circ} \subseteq S^{\circ}$
(\circ contravariance)	$(R \cdot S)^{\circ} = S^{\circ} \cdot R^{\circ}$
(left modular)	$(R \cdot S) \cap T \subseteq R \cdot (S \cap (R^{\circ} \cdot T))$

DEDEKIND VARIATIONS

(left modular)	$(R \cdot S) \cap T \subseteq R \cdot (S \cap (R^{\circ} \cdot T))$
(right modular)	$(R \cdot S) \cap T \subseteq (R \cap (T \cdot S^{\circ})) \cdot S$
(weak distr)	$(R \cdot S) \cap T \subseteq (R \cap (T \cdot S^{\circ})) \cdot (S \cap (R^{\circ} \cdot T))$

\cap LAWS

(\cap associativity)	$(R \cap S) \cap T = R \cap (S \cap T)$
(\cap commutativity)	$R \cap S = S \cap R$
(\cap idempotence)	$R \cap R = R$
(\cap abbreviation)	$R \subseteq S \equiv R = R \cap S$
(\cap cancellation)	$R \cap S \subseteq R, R \cap S \subseteq S$
(\cap left fusion)	$T \cdot (R \cap S) \subseteq T \cdot R \cap T \cdot S$
(\cap right fusion)	$(R \cap S) \cdot T \subseteq R \cdot T \cap S \cdot T$

◦ LAWS

(reduction)	$R \subseteq R^\circ \equiv R = R^\circ$
(wrap)	$R \subseteq R \cdot R^\circ \cdot R$
(dist over \cap)	$(S \cap R)^\circ = R^\circ \cap S^\circ$

MISC

(Dedekind: $T = id, R = R^\circ$)	$(R^\circ \cdot S) \cap id \subseteq (R \cap S)^\circ \cdot (R \cap S)$
(Dedekind: $S = id$)	$R \cap T \subseteq R \cdot (id \cap (R^\circ \cdot T))$

KERNEL LAWS

(definition)	$\ker R = R^\circ \cdot R$
(duality)	$\ker R^\circ = \text{img } R$
(monotonicity)	$R \subseteq S \Rightarrow \ker R \subseteq \ker S$
(symmetry)	$(\ker R)^\circ = \ker R$
(intro)	$R \subseteq R \cdot \ker R$
(\cdot kernel)	$\ker (R \cdot S) = S^\circ \cdot \ker R \cdot S$
(\cap kernel)	$R^\circ \cdot S \cap id \subseteq \ker (R \cap S)$

IMAGE LAWS

(definition)	$\text{img } R = R \cdot R^\circ$
(duality)	$\text{img } R^\circ = \ker R$
(monotonicity)	$R \subseteq S \Rightarrow \text{img } R \subseteq \text{img } S$
(symmetry)	$(\text{img } R)^\circ = \text{img } R$
(intro)	$R = \text{img } R \cdot R$

ORDER TAXONOMY

(reflexive)	$id_A \subseteq R$
(coreflexive)	$R \subseteq id_A$
(transitive)	$R \cdot R \subseteq R$
(symmetric)	$R \subseteq R^\circ$
(anti-symmetric)	$R \cap R^\circ \subseteq id_A$
(connected)	$R \cup R^\circ = \top_A$
(entire)	$id \subseteq \ker R$ (total relation)
(simple)	$\text{img } R \subseteq id$ (partial function)
(surjection)	R° entire
(injection)	R° simple

COREFLEXIVES

(symm & transitive)	$R = R^\circ = R \cdot R = R \cap id$
(cancellation)	$R \text{ coreflexive} \Rightarrow (R \cdot T \subseteq T) \wedge (T \cdot R \subseteq T)$
(distributivity)	$R \text{ coreflexives} \Rightarrow (R \cdot T) \cap S = R \cdot (T \cap S)$